

A Factorial Study of Adolescent Thought Using Piaget Type Tasks

THESIS
Submitted to
THE UNIVERSITY OF RAJASTHAN, JAIPUR
for the award of the degree of
DOCTOR OF PHILOSOPHY
in
EDUCATION
October, 1980

Supervised by :
Dr. N. VAIDYA
Professor and Head
Department of Education

Submitted by :
TEG SINGH SANDHU

REGIONAL COLLEGE OF EDUCATION
AJMER (RAJASTHAN)

Certificate

Date

Oct. 15, 1930

Dr. H. Vaidya
Professor and Head,
Department of Education,
Regional College of Education,
Ajmer (Rajasthan)

I am pleased to certify that Mr. Jog Singh
Tandhu has worked on the problem "Pastorial
Study of Adolescent Thought Using Piaget Type
Tasks" under my supervision. This thesis is
his original work and he is submitting it for
the award of Ph.D. degree. It is further
certified that no part of the work has been
submitted for any degree earlier.

Waidy
(H. VAIDYA)

Acknowledgements

I owe first of all a sense of deep gratitude to my learned supervisor, Dr. N.Valiyya, Professor and Head of the Department of Education, Regional College of Education, Ajmer, for his worthy guidance regarding the present study. I feel very much grateful to him for the award of a research fellowship to work on an ERIC (NCERT) project under his supervision, which enabled me to carry out this study conveniently. I am also thankful to him for permitting me to use a part of the data collected.

I am very grateful to Dr. T.K.Mitra, Director, NCERT; Dr. B.N.Mukherjee, Director of Research, Council for Social Development; Dr. B.C.Das, Professor and Head of the Department of Teacher Education, NCERT; Dr. R.C.Misra, Professor and Head of the Department of Measurement and Evaluation, NCERT; and Dr. R.N.Mehrotra, Principal, Central Institute of Education, New Delhi, for devoting their valuable time to discuss the frame-work of this study.

I am indebted to the Indian Council of Social Sciences Research, New Delhi for sanctioning a grant for computer analysis of the data. I am grateful to the Council for Social Development, New Delhi, for providing me all facilities for the analysis of the data. I am especially thankful to Mr. B.C.Nagi for taking personal interest in my investigations. I also owe my thanks to the Computronics India, New Delhi, for extending co-operation and the services of the Computer.

I should also thank Dr. J.K. Sood, Dr. G.M. Bhardwaj and Dr. I.M. Bahl who helped me a lot, not only in the execution of this study but also took keen interest in solving my personal problems during my stay at Regional College of Education, Ajmer.

My thanks are also due to the heads, teachers and students of all the schools who offered me their full co-operation in the process of data collection. Particularly, I would like to mention Mrs. Inderjit, Headmistress, Govt. Co-educational Model High School, Parsa Mahna (Paridhot) for helping me to contact the various schools. I am thankful to Mr. Gian Singh and Mr. Rajinder Singh Mann for helping me in preparing the manuscript. Finally, I should like to thank Mr. N.K. Gupta for typing the manuscript with best accuracy and speed.

Teg Singh Sandhu
(TEG SINGH SANDHU) 15/10/80

Table of Contents

Chapter		Pages
I	Some Background Issues Underlying the Problem	1 - 10
	Introduction	
	Thinking: A Multi-dimensional Activity	
	Philosophical Bases of Thinking	
	Philosophical Theories of Thinking	
	Psychological Bases of Thinking	
	Psychological Theories of Thinking	
	Solving the Problem	
II	Adolescent Thought as Viewed by Jean Piaget	10 - 31
	Introduction	
	The Stages of Cognitive Development in relation to the Characteristics of Concrete-Operational and Formal-Operational Thought	
III	Adolescent Thought : A Critical Review	32 - 50
	Introduction	
	Studies on Stages of Development	
	Studies Regarding the Relationship of Formal Thought with Age	
	Studies Related to Sex Differences	
	Studies on Relationship Between Formal Thought and Intelligence	
	Studies Regarding the Relationship Between Formal Thought and Culture	
	Studies on Academic Achievement, Personality and Adjustment in Relation to Formal Thought	
	Concluding Statement	
	Status of Research on Adolescent Thought	
	Problems Posed in the Field	
	Distinguishing Characteristics of the Present Study	

Section A

Frame Work of the Study

Introduction

Hypotheses

Sample

Data Collection Schedule

Statistical Treatment of the Data

Section B

Description of the Test
of Piaget Type TasksReliability and Validity of the Test
of Piaget Type Tasks (Task-wise)Reliability of the Test of Piaget
Type Tasks (Combined)Validity of the Test of Piaget
Type Tasks (Combined)

Section C

Description of the
Other Tests UsedCattell's Culture Fair
Intelligence Test (Scale 2)

Jalota's General Mental Ability Test

Dubey's Reasoning Ability Test

Asthana's Adjustment Inventory

Cattell's High School Personality
Questionnaire (THQ)

Space Relations Test (SAT)

Academic Achievement in School Subjects

Section A

Results of Descriptive
Statistics

Section II

Results of Bivariate Analysis

Interpretations and Discussion

VI	analysis of Mathematical Structure Underlying the Adolescent Thought	125 - 130
----	---	-----------

- Introduction
- Some Background Studies
- The Present Study
- Correlation Matrix
- Obtaining the Factors
- Interpretation of Factors
- The Current Picture of the Structure
of Adolescent Thought
- Concluding Statement

VII	Imp Effect Encountered : An Observation of Second Interest	130 - 163
-----	---	-----------

VIII	Summary and Conclusions	164 - 175
------	-------------------------	-----------

- Educational Implications
- Problems for Further Research

Bibliography

Appendices

- i) Test of Piaget Type Tasks (Part I)
- ii) Test of Piaget Type Tasks (Part II)
- iii) An Analogy Between Piagetian Grouping
of Thought and Group Theory in Algebra
- iv) Code Sheet of Variables
- v) Imp Effect as Observed During
Problem Solving
- vi) Original Data of Different Variables

List of Tables

Table No.		Page
1.	Coefficients of Reliability and Validity of the Test of Piaget Type Tasks (Task-wise)	77
2.	Coefficient of Reliability of the Test of Piaget Type Tasks (Combined)	78
3.	Validity of the Test of Piaget Type Tasks (Combined) against Verbal Intelligence	80
4.	Validity of the Test of Piaget Type Tasks (Combined) against Non-verbal Intelligence	81
5.	Validity of the Test of Piaget Type Tasks (Combined against Reasoning Ability	82
6.	The Values of Mean, Median, Mode, Standard Deviation, Standard Error, Kurtosis and Skewness for Fourteen Dimensions of Personality (BPQ)	89
7.	The Values of Mean, Median, Mode, Standard Deviation, Standard Error, Kurtosis and Skewness for Five Measures of Academic Achievement	91
8.	The Values of Mean, Median, Mode, Standard Deviation, Standard Error, Kurtosis and Skewness for the Measures of Adjustment, Verbal Intelligence, Non-verbal Intelligence, Space Relations and Reasoning Ability	92

9.	The Values of Mean, Median, Mode, Standard Deviation, Standard Error, Kurtosis and Skewness for the Ten Measures of Adolescent Thought	94
10.	F-ratios for the Analysis of Variance with respect to Five Age Levels and the Performance on Each of the Ten Piaget Type Tasks	97
11.	Mean Score and Standard Deviation of Each Task at Different Age Levels	99
12.	Results Regarding the Comparative Performance of Boys and Girls on the Task of Classification at the Five Age Levels as well as for the Combined Groups	102
13.	Results Regarding the Comparative Performance of Boys and Girls on the Task of Grouping of Thought at the Five Age Levels as well as for the Combined Groups	103
14.	Results Regarding the Comparative Performance of Boys and Girls on the Task of Generalization: Arithmetical and Algebraic Symbols at the Five Age Levels as well as for the Combined Groups	104
15.	Results Regarding the Comparative Performance of Boys and Girls on the Task of Permutations and Combinations at the Five Age Levels as well as for the Combined Groups	105

16.	Results Regarding the Comparative Performance of Boys and Girls on the Task of Ratio and Proportion at the Five Age Levels as well as for the Combined Groups	106
17.	Results Regarding the Comparative Performance of Boys and Girls on the Task of Probing Questions at the Five Age Levels as well as for the Combined Groups	107
18.	Results Regarding the Comparative Performance of Boys and Girls on the Task of Interpretation and Coordination of Information at the Five Age Levels as well as for the Combined Groups	108
19.	Results Regarding the Comparative Performance of Boys and Girls on the Task of Stating and Testing Hypotheses at the Five Age Levels as well as for the Combined Groups	109
20.	Results Regarding the Comparative Performance of Boys and Girls on the Task of Space Visualization at the Five Age Levels as well as for the Combined Groups	110
21.	Results Regarding the Comparative Performance of Boys and Girls on the Task of Grasping the Essence of the Problem at the Five Age Levels as well as for the Combined Groups	111
22.	Significant ⁸ t-differences Between the Performance of Boys and Girls on Ten Piaget Type Tasks at the Five Age Levels as well as for the Combined Groups	114

23.	Coefficients of Correlation Between the Performance on Piaget Type Tasks and the Other Variables	116
24.	Correlation Matrix Involving Ten Piaget Type Tasks	131
25.	Original Factor Matrix showing Factor Loadings of Piaget Type Tasks on the Significant Factor Extracted	132
26.	Correlation Matrix (34 x 34)	135
27.	Original Factor Matrix	137
28.	Varimax Rotated Factor Matrix	138
29.	Mean Scores and Standard Deviations on the Dimensions of Ratio and Proportion, Grasping the Essence of the Problem and Space Visualization at Different Age Levels	161

List of Figures

Fig. No.		Page
1.	Twenty-nine Geometrical Figures Consisting of Triangles, Squares and Circles of Different Sizes, Colours and Shapes	59
2.	Hierarchical Classification of Students	64
3.	Beakers Containing Different Colourless Chemical Reagents	66
4.	The letter 'V' in Two Different Sizes	68
5.	A Board having Nine Squares Traced on It	71
6.	A Simple Pendulum	73
7.	Various Sets of Nine Dots	74
8.	Graphs showing Performance on Different Piaget Type Tasks in Relation to Age During Formal-Operational Period	100
9.	Significant Factor Loadings of Different Variables on Original and Varimax Factor I	139
10.	Significant Factor Loadings of Different Variables on Original and Varimax Factor II	142
11.	Significant Factor Loadings of Different Variables on Original and Varimax Factor III	144
12.	Significant Factor Loadings of Different Variables on Original and Varimax Factor IV	145
13.	Significant Factor Loadings of Different Variables on Original and Varimax Factor V	147

14.	Significant Factor Loadings of Different Variables on Original and Varimax Factor VI	149
15.	Significant Factor Loadings of Different Variables on Original and Varimax Factor VII	150
16.	Significant Factor Loadings of Different Variables on Original and Varimax Factor VIII	152
17.	Group Effect as Observed Regarding the Performance on the Task of Ratio and Proportion	162
18.	Group Effect as Observed Regarding the Performance on the Task of Space Visualization	162
19.	Group Effect as Observed Regarding the Performance on the Task of Grasping the Essence of the Problem	162

CHAPTER - I

Some Background Issues
Underlying the Problem

CHAPTER I

Some Background Issues Underlying the Problem

Introduction

Human thinking is very complex. Our present day scientific and literary culture is the product of this thinking. The investigation of the processes of thinking is very difficult, intriguing and mysterious. No doubt, thinking in general has been the subject of study of many philosophers and psychologists since long but it is only in this century that the experimental science began to interest itself in the processes and the structure of human thought. Piaget has studied the developmental aspect of human thinking starting from birth till late adolescence and has established an invariant sequence of four stages of development, i.e., the sensory-motor period (birth to 2 years), the pre-operational period (2 to 7 years), ^{the}concrete-operational period (7 to 11 years) and the formal-operational period (11 to 15 years). Though a good work has been done on the earlier stages of development, very few attempts have been made to investigate the adolescent thought, i.e., the formal-operational period and that too in a very narrow range.

Since the development of formal-operational thought is very important for the cognitive functioning of an individual throughout his life, and also it is most crucial from the educational point of view, the important issue before us today is to identify the structure of adolescent thought taking into

account the maximum number of its dimensions. Equally important is to develop a matching model of curriculum and pedagogy for class-room instruction in different subjects.

Before discussing in detail the Piagetian stages of development and their characteristics, let us have a general look on thinking and its philosophical and psychological bases.

Thinking : A Multi-dimensional Activity

There are very few people who ever think about thinking. Those who have traditionally given descriptions of what a man does who thinks, have been the philosophers. They have chiefly relied on their own personal experience as their data. It is only recently that psychologists have tried to find out what happens when we think and what conditions influence our performance by applying the methods of science to this human capacity. Neither in philosophy nor in psychology have the results of this abstract entity been really successful in the past. But now a days, a general consensus has developed among the research workers and philosophers that thinking is much more complicated business than common-sense acquaintance with the term. According to the *Encyclopaedia Britannica* :

The term thinking itself has many definitions, no one of which is satisfactory to everyone. A useful one for those who attempt to study it scientifically defines thinking as that aspect of human (and animal) activity that primarily involves processing of information ... The operations or processes involved in thinking are many and varied in kind and complexity. They include the simpler logical operations such as matching two items or substituting one for another,

and also many complex ones. They include the simpler mathematical operations such as addition and multiplication, and also more complex ones such as differentiation and integration. And they include many less logical processes. These processes are themselves the result of learning and hence vary from one thinker to another.

Thus thinking is a multi-dimensional activity with respect to both processes and products.

Philosophical Bases of Thinking

Thinking is an essentially human activity occurring in two basic forms. We may think in order to attain knowledge of what is, what must or what may be the case. We may also think with a view to making up our mind about what we will or what will not do. Following Aristotle, these two forms of thought may be called contemplation and deliberation respectively. Both forms may be carried on well or badly, successfully or unsuccessfully, intelligently or unintelligently. When contemplation is successful, it terminates in a conclusion and successful deliberation terminates in a decision or resolution. The form of reasoning involved in contemplation may be called theoretical and the form involved in deliberation may be called practical (Aune, 1967). The thoughts involved in both contemplation and deliberation have the following basic features:

- a) They are characteristically, but perhaps not necessarily, carried on in foro interno.

b) They are directed toward an object or a number of objects, and they either attribute something to, or deny something about this object or number of objects.

c) The language used to describe them is non-extensional in the sense of possessing at least one of the three intentional marks, i.e. :

i) Neither the sentence nor its negation implies either the existence or the non-existence of that thing to which the substantive expression truly applies.

Example : Nam is thinking about Ghosts.

ii) A non-compound sentence about thinking may contain a propositional clause in such a way that neither the sentence nor its negation implies either the truth or falsity of the propositional clause.

Example : It occurred to Mohan that earthquakes cause heart attack.

iii) Although things or events have many names and may be described in many different ways, the fact that a person thinks of them in connection with one name or description does not imply that he thinks of them in connection with some other name or description.

Example : Cohen thought that the author of Gita wrote Ramayana.

d) Thoughts are often conceived in relation to and are felicitously expressible by, specific verbal forms, i.e., they are often essentially linguistic or conceptual.

e) Particular thoughts have some kind of logical form; they may be categorical, hypothetical, disjunctive, universal, particular and the like.

When we investigate the processes of thinking, philosophically speaking, we must be aware of the features mentioned above regarding the construction of the text, i.e., how and to what extent can it possess these features.

Philosophical Theories of Thinking

A survey of the full range of views on thinking that have been influential in the history of philosophy, could reveal that most important theories of thinking have been variants of one or more of the following basic views : Platonism, Aristotelism, Conceptualism, Imagism, Psychological Nominalism, Behaviourism, 'Iyies' Approach and Analogy Theory.

Platonism

According to Platonism, thinking is either a dialogue in the soul involving mental words that refer to forms (such as Redness, Triangularity, Flying, etc.) and possibly to individuals (such as Socrates, etc.) or a spiritual activity of inspecting or recollecting forms and discerning their nature and inter-relations.

Aristotelianism

According to Aristotelianism, thinking is an act of the intellect in which a thing's essence or intelligible form actually qualifies the intellect; to think about humanity is for one's intellect to be informed literally of the essence of humanity. For instance, humanity involves animality meaning

that one's intellect is also informed by the other essence, the latter being also the part of the former.

Conceptualism

For the conceptualists, thinking is an activity of bringing concepts or ideas before the mind, these being either innate or else formed by abstraction from sense experiences and thus actually sharing the abstract features of those experiences

Imagism

For the Imagists, thinking is basically a sequence of episodes involving images. These images are tied to certain "habits", which are the inveterate tendencies of the mind to move from one image to another. To think about triangularity, according to this view, is to imagine some particular triangle while disposed to pass on to other images of the same sort.

Psychological Nominalism

According to the Psychological Nominalists, thinking is literally a dialogue in the soul (or better in the head) involving the use of verbal images or mental words which denote things or classes of things. In this view a complete thought is a mental utterance of a sentence, such as "Tom is tall".

Behaviourism

According to Behaviourists, thinking is either thoughtful overt speech - thoughtful in the sense that it is in accordance with the various principles of relevance. Thinking is also defined by this school as evidence or inference that the agent

is prepared to cite in explanation of his behaviour - or a changing series of dispositions to behave intelligently that the agent can at any time avow.

Ryle's Approach

According to Ryle, the idea that a non-habitual intelligent behaviour is always guided by silent thought, is mistaken. In his opinion, reference to interior and exterior acts of thinking is not in anyway needed for the explanation of most intelligent behaviour. The verbal behaviour may be regarded as intelligent, thoughtful, and even rational if it is done in accordance with certain principles of inference, evidence, relevance, etc. Therefore, purely overt calculation or deliberation is itself a process of thinking and that thinking is not something that is necessarily done silently in the soul. In other words, overt thinking is just as useful a mode of thinking as any other, and there is no need, even no point, in always hunting for hidden acts of thought.

Analogy Theory

Although Ryle's view of thinking does not, as a whole, succeed, it does come close to the truth. As all the calculation or deliberation that accounts for a man's actions is not done loud or on paper, so the reference to silent thought is constantly and legitimately made in order to account for activities that would otherwise remain inexplicable. According to Analogy Theory, a man may make a move in chess after sitting in silent anguish for long minutes at the board; and the

intelligence of this move will remain a stubborn question mark until perhaps after the game, he outlines the strategy behind it. Ryle has also argued that a man can learn to mutter to himself as well as to mutter out loud. While silent thought need not be inner speech, it may still be an activity that is at least formally analogous to speech. Thus while the thought of p is empirically different from the act of p , it may still be regarded as formally the same: both are activities that conform to the same principles and have many of the same implications. What is essential in both cases is that formally analogous activities are carried on in accordance with the same basic principles. This theory of thinking does more than merely correct the shortcomings of Ryle's view and it is perhaps the most satisfactory account of thinking yet developed by philosophers.

Psychological Bases of Thinking

Psychologically speaking, thinking is defined operationally as the establishing of order(s) in the apprehended world. This ordering relates to objects as well as to representations of the world of objects, and the ordering of relations between the representations of objects. The figurative or pictorial representation (imagery) of what has been perceived makes it possible to order according to equality, similarity or difference. The objects with the same visual, acoustic, haptic or kinesthetic qualities are treated as belonging together, inequalities lead to separation from the grouping of similar objects. The action of ordering with figurative and

pictorial images is called intuitive thinking. Thought is said to be intuitive if the ordering of the experienced world takes place according to states conditioned by feeling or motivation. Thought arbitrarily links persons, things or objects coinciding fortuitously with these inner states. If wish-fulfilment tendencies determine the results of thinking, the thought processes are defined as primary. On the other hand, when rational ordering techniques determine the results of thinking thought processes are defined as secondary. Magical thinking orders the relations of image, sign or symbol to the object as if objects as well as representations of them were capable of acting like human beings. This way of thinking is frequently found in younger children, uninformed adults and in exceptional existential states.

If the representations, thoughts, and their relations being ordered can no longer be expressed in imagery or figuratively, then thinking is non-intuitive, abstract or conceptual. In such thinking activity the task determines the direction that thinking will take. In the further course of socio-historical development it becomes possible to abstract from real actions, to replace them with mental actions, or those described with words (different signs are also used for this purpose). On this foundation the highest-abstract and generalized form of thinking arises. Simultaneously, a separation of cognition takes place and it becomes a special theoretical activity, which nevertheless remains linked to practice as the source and

criterion of accuracy, and the place where the results of thinking will be used.

It is only recently that psychologists have tried to find out what happens when we think and what conditions influence our performance by applying the methods of science to this human capacity. Though they are not fully successful in this venture, yet, they have shown that thinking is a much more complicated ^{speaking,} affair than common-sense acquaintance with it. Psychologically / thinking may be considered as an active purposeful process of cognition - a search for solutions to practical and subsequently to theoretical problems.

Psychological Theories of Thinking

In the psychological theory and research, the term 'thinking' has acquired a restricted meaning and has become identified with problem-solving. Thinking starts when we become confronted with some perplexity or problem. As long as things are going smoothly there is no necessity for thought. Dewey (1910) cites the homely example of a man progressing without difficulty along a road until he comes to a fork in it. The emphasis on thinking as problem-solving is very much in agreement with the psychological interpretations of it and also the theoretical traditions are equally committed to viewing thinking as problem-solving. For instance, the psychology of thinking based upon learning theory assumes that the same basic concepts of stimulus, response, discrimination and generalisation are

applicable to problem-solving and that thinking is to be conceived as part of the process by which an organism adapts to its new environment. The greatest merit in this definition is that it relates the process of thinking to the behaviour of man coping with their environment which is what we call experimentation or learning. Thinking, in psychological literature, may be reviewed from the viewpoints of the five major schools of thought - Behaviourism, Gestalt, Functionalism, Psychoanalysis and the Geneva School. A brief introduction of these schools of thought has been given below:

Behaviourism

As a school of psychology, behaviourism is thought of as originating with Watson who announced the behaviourist position first in 1913 and thereafter became its vigorous spokesman. His other contemporaries were : Thorndike, Pavlov, Guthrie, Tolman and Hill. However, Skinner also joined the field later on. Most of them conceived themselves as biologists who happen to be interested in how organisms behave under various circumstances. They have preferred, usually, experimentation on animals and infants. Behaviourists have a conviction that a science of psychology must be based upon a study of that which is overtly observable. According to them, the behavioural event (thinking) begins with stimulation provided by the external world and ends with a response while the environment plays a part inbetween.

Gestalt Theory

This school of thought developed in Germany with Prof. Max Wertheimer as its founder. Wolfgang Kohler and Kurt Koffka being the other associates of Wertheimer made the theory popular in America through their visits and books. Gestaltists begin with abstract ideas, concerning the nature of perception, thinking and the structure of psychological experience, and then they proceed to interpret familiar observations in terms of these novel concepts. Insight concept is one of the main contributions of the Gestalt school. There is a general agreement that insight occurs when there is integration of experience - a restructuring or seeing of a new relationship to the problem at hand.

Functionalism

The idea of the programs of functionalism was first propounded by Titchener, who later on turned out to be an opponent of it. To put forward the concept of functional psychology according to which mind may be regarded as the collective name for a system of functions of the psychophysical organism. Later on many other psychologists became interested in the function of the mind as it is used in adaptation of the organism to its environment. University of Chicago (1900-1930) was the mainland where functionalism rose to its prominence. The psychologists primarily responsible for the growth included James Angell and Harvey Carr in addition to John Dewey. According to them, functionalism is a psychology of the adjustment of the organism to its environment. It is a cause-and-effect

psychology which is interested in how, why and what of mental operations. At the same time it studies the physiological substratum of mental events (thinking). John Dewey, saw the relevance of psychology to classroom teaching and learning. Later on functional psychologists like Francis Galton and James Cattell became interested in differential mental abilities, what they were good for, and how to measure them with mental tests.

Psychoanalysis

The discussion on thinking would be incomplete without reference to Sigmund Freud as he has influenced the whole field of psychological thinking very widely. Though it will be difficult to state the postulates of thinking in the propositional form from the psychoanalytical viewpoint as the theory is too complex, however, the general standpoints can be discussed. Writers on psychoanalysis often stress that it is genetic as well as a dynamic theory, i.e., continuities in the life of the individual deriving from the past, leave their impact upon what is happening in the present. The theory suggests that a very young child is usually susceptible to influences which leave a permanent mark on his personality. According to the psychoanalytic theory, adult thinking either oscillates between or combines the two modes of thinking. The primary process thinking which is impulse-driven and largely irrational seeking immediate gratification at all costs even by hallucinations, and the secondary process thinking which is patient and logical willing to postpone gratification for the

future gains. The psychoanalytic theory has helped to erase the boundaries between the neurotic and the normal so that what was once relegated to as abnormal psychology has now become a part of general psychology. Finally, the genetic or the developmental aspects of psychoanalysis have brought to the fore the need for an adequate ego psychology.

The Geneva school

Jean Piaget and his collaborators in Geneva have produced abundant observations, both naturalistic and experimental, in support of his theory. Piaget chooses problems for investigation from the area of cognition without considering at the same time any other outside variable. He acknowledges his debt to Gestalt psychology in his thinking. Gestalt psychology is quite rich in ideas but Piaget goes a step further when he says that his schemas are more dynamic and modifiable structural units than the "gestalts". The schemas are characterized by mobility, transposability, generalisability, elasticity, self-modifiability to fit new data, built-in activity, and lastly they undergo evolution through corrective controls. According to Piaget, the inferior schemata slowly become superior ones and comparatively speaking more adequate to reality adaptation. In his theory, there is no place for insight because the complex schemata arise or evolve from the simpler ones already formed.

Through the use of his symbolic logic, Piaget has been able to point out the properties of thinking at various age levels in terms of the thinking operations of children within

an age group which they are capable or incapable of performing. Thus, as the child moves from middle childhood to adolescence his thought processes move from concrete operations to formal propositional thinking. The thought processes characterize the scientific method: considering all possibilities, making 'if-then' hypotheses subject to verification, organising the principles into some sort of network. It is worth mentioning here that language also plays a major part through permitting the child to represent action in thought. It does not produce the evolutionary stages of intelligence but is instead an agent in the service of intelligence.

The above mentioned discussion on thinking in its multi-variate aspects is of varying relevance to the researchers trained in different schools of thought. The starting thread of thought can be initiated in any context but the problem becomes uncontrollable when all the contexts are considered simultaneously. Secondly, it is beyond the reach of any single investigator to tackle this problem in all its aspects because of several intellectual complexities, scarcity of literature, paucity of tools and data and lastly, the physical limitations of time and other resources. Currently, the Geneva school has presented a fruitful model in which specific research on formal thought is badly needed. Consequently the problem has been posed as follows:

Posing the Problem

Now a days the major interest of psychologists and educationists is not only to understand individuals but also to

study the general trend of the development and structure of the human mind. The scientific investigation of thinking processes and of the structure of human mind is gaining importance because the growth of a highly logical mind has become one of the most important goals of educational instruction in the modern scientific society. The social scientists are expected to study the processes of thinking and its structure rationally as the physicists study the atom. At present whereas the products of thought in the form of our civilisation and culture are well known there is very little knowledge available about the underlying processes of thinking or about the very nature of thinking itself. In the contemporary usage the term thinking refers to a wide range of mental exercises, i.e., abstracting, analysing, knowing, opining, comparing, guessing, imagining, judging, reasoning, recalling, recognising, reflecting, remembering, searching for conclusions and understanding, etc. A late trend that has emerged in psychological literature considers thinking somewhat akin to problem solving (Valdya, 1976). A good work has been done on this approach by many research workers including Jean Piaget.

Jean Piaget has contributed immensely to the whole field of psychology in general, and to modes of human thinking particularly, over a period of 60 years or so. The important feature of Piaget's work is that he is more interested in studying the structure of developing human mind than its function and content. Piaget postulates the existence of cognitive structure which

like content and unlike function, does indeed change with age, and these developmental changes constitute the major object of study for him. According to him structures are the organizational properties of intelligence, created through functioning and invariable form of behavioural contents whose nature they determine.

As pointed out by Inhelder & Piaget (1958) "It is surprising that in spite of the large number of excellent works which have been published on the affective and social life of the adolescent so little work has appeared on the adolescent's thinking". No doubt thinking in general has been studied from the various standpoints, such as, Thematic thinking, Explanatory thinking, Productive thinking, Integrative thinking and Problem-solving thinking, etc., by many philosophers and psychologists like Dewey, 1910; Freud, 1940; Burt, 1940; Wertheimer, 1945; Humphrey, 1951; Keats, 1955; Bruner et al, 1956; Ruswell, 1956; Bartlett, 1958; Mills and Dean, 1959; etc. Most of the investigations have been done developmentally. In the psychological literature there is a dearth of studies which could have analysed mathematically the content and form of the adolescent thought comprehensively.

The present study was designed to investigate the adolescent thought using the tests of Piaget Type Tasks and other variables, such as, intelligence (verbal and non-verbal), reasoning ability, space relations, academic achievement,

adjustment and other personality traits, and to analyse mathematically the contents of adolescent thought to identify the underlying structure of the same. It also attempted to find out, at the same time, the relationship between the measures of the dimensions of adolescent thought and the independent variables, i.e., age, sex, intelligence, reasoning ability, space relations, academic achievement, adjustment and personality traits.

CHAPTER II

Adolescent Thought as Viewed
by Jean Piaget

CHAPTER II

Adolescent Thought as Viewed by Jean Piaget

Introduction

Feeling that the approaches of philosophers to the problems of thinking were far too speculative and not sufficiently experimental, Piaget decided to spend four or five years for studying the development of logical thinking in the child. This interim study has turned out to be his life's work spanning a period of about sixty years. Piaget's work during this period has taken many forms and has been concerned with diverse problems but there is a remarkable continuity in his research which originates from his initial interest in problems of biology and the philosophy of knowledge, and constitutes a dialectic underlying all the theory and research. On the one hand there are the facts pertaining to man as a physical organism - the facts of biology and neurophysiology which indicate that man is equipped with a certain physio-chemical structure and that he must adapt to his environment in order to survive-while, on the other hand, are the constructions of man the thinker, i.e., the social and physical sciences themselves : logic, mathematics, social conventions and laws. The philosophical problem which arises from the confrontation of these two sets of data is : how do we formulate the relationship between human achievements, which include scientific laws and generalizations and those conditions which science asserts to determine our experience and behaviour? The research which Piaget has undertaken represents an attempt to answer this question (the question of the nature and

status of human knowledge) by experiment and observation .
 Piaget provides a larger content in which to view the acquisition of knowledge and competence as a consequence of growth and interaction with the physical and social environment. Piaget (1970a) notes that classical theories of development consider three aspects:

1. Biological maturation.
2. Experience with physical environment.
3. Experience with social environment.

He adds the fourth consideration also, while explaining his own viewpoint about the above three:

4. Equilibration.

The fourth consideration which makes Piaget differ from the classical theorists, needs further explanation.

Equilibration

Piaget believes that equilibration, if understood as he means it, is the fundamental factor in development, and necessary to coordinate the other three factors. Equilibration is a progressive, self-regulating process which leads step by step to a final state of reversibility that characterizes higher cognitive structures. It makes the child move from static configuration to the notion of a transformation, and once the child's thought includes the concept of a transformation he is prepared for the next stage. Piaget has described conservation also as an invariance in the midst of transformation which illustrates reversibility. Two additional concepts are needed in order to understand what Piaget means exactly by equilibration - these are

assimilation and accommodation. Assimilation is the first part of the two-part process of interaction between external reality and the child's own attained cognitive structure. Piaget uses the term assimilation for the process through which the new environmental experiences fit - in or become a part of the existing cognitive organization of the child. Accommodation is the second part of the process which deals with the change in the child's cognitive schema or structure so as to conform to the new external reality.

The Stages of Cognitive Development

Piaget does not see the process of cognitive growth merely as a matter of continuous and quantitative improvements which remain qualitatively constant throughout the life span of human beings. He considers the qualitative changes in the underlying processes as a fundamental fact of mental growth. He has grouped these qualitative changes into a succession of four global stages. These stages satisfy a set of criteria of which the following are the most important : (a) qualitative change in cognitive contents, (b) a culturally universal invariant sequence in the overall progression of stages, (c) inclusion of the cognitive structures of each preceding stage in each subsequent stage, and (d) an overall integration of the structures of each stage. The description of the stages of development has been presented below:

1. The sensory-motor period (birth to 2 years)
2. The pre-operational period (2 to 7 years)
3. The concrete-operational period (7 to 11 years)
4. The formal-operational period (11 to 15 years).

The Sensorimotor Period

It is the period of about 18 months which begins with a stimulus for a few minutes, and ends when language and other symbolic ways of representing the world first appear. The primary information about this, however, is scattered over a number of Piaget's books and articles, yet major accounts are the empirical data regarding the sensorimotor period are available in the *Intelligence in Children* series (1951, 1953 and 1954). The series was originally published in French between 1936 and 1943. The first book (1951) describes the development of intelligence, play and conceptual evolution during the early years. The second book (1953) deals with the general characteristics of sensorimotor development. The third book (1954) is concerned with the child's intellectual grasp of space, time, causality and objects. Piaget has divided the overall developmental sequence of sensorimotor period into six stages. At the first stage (birth-1 month) the child shows little besides the reflexes with which he is provided at birth. At the second stage (1-4 months) the various reflex activities begin to undergo modifications with experience, and to inter-coordinate one experience with the other. At the third stage (4-8 months) the infant begins to perform actions oriented towards objects and events outside and beyond his own body. At the fourth stage (8-12 months) there is definite intentionality manifested by the means-ends or instrumental action sequences. Only familiar or habitual behaviour is involved. At the fifth stage (12-18 months) the child experiments to find new means,

and to pursue novelty for its own sake. At the sixth stage (18-24 months) the child begins to make internal and symbolic representations of sensory-motor problems and to 'invent' solutions by implicit rather than explicit trial and error behaviour. Thus Piaget's sensory-motor stage is a step-by-step account of the infant's progress from cognitive contents that are reflexive, self-centred and disorganized to cognitive contents that are instrumental, adapted to the demands of the environment and well organized. The remarks of Piaget (1954) that 'intelligence organizes the world by organizing itself' demonstrate the phenomenon well. It is also worth noting as Bolton (1972) remarked that 'intelligence organizes itself by organizing the world.' Thus, it is the organization of the intelligence as well as of the world which takes place during the sensory-motor period.

The Pre-operational Period

The child starts imitating the behaviour of the object at the sixth substage of sensory-motor period. These exterior imitations on the part of the child become internalized as 'images' - an image being a covert reproduction of an initially overt accommodation. In the pre-operational period, as the name implies, the internalization of actions has not reached the stage in which the child can make use of a system of operations. The development in this period can be conceived as preparing the way for this achievement through the increasing co-ordination of assimilation and accommodation in the child's symbolic activities. Piaget uses the word 'schemas' for the

imitative accommodation that is focussed on the outline properties of an object. Mostly, the child's thinking is dominated by his perception which gives rise to the limitations of the pre-operational thought. According to Piaget, the child at the pre-operational stage fails to conserve mass, weight, length, volume, etc. Thus, the child is not able to perform successfully on different invariance problems. Piaget has described that the pre-operational child reveals the lack of reversibility in his schemes when he fails on the invariance problems. The child does not realize that the operation can be reversed to restore the original equality. Also the pre-operational child has difficulties in understanding the effect of different points of view on the same event and in the integration of temporarily separate bits of information. Since the pre-operational period (2-7 years) is a long one in which many changes gradually occur, ~~whereas~~ Piaget has divided this period into two parts :

- a) Ranging from 2 to 4 years.
- b) Ranging from 4 to 7 years.

At the sub-stage (a) the child fails to construct hierarchical arrangements because after a short while he forgets the defining properties. The sub-stage (b), i.e., the period from 4-7 years, has been labelled as the intuitive stage. During this period the child acquires a mode of dealing with many of the problems of integrating different viewpoints and information from

different sources. Even though the child can feel his way frequently to a correct answer through a problem but he still does not have a clear conceptual representation.

Thus, the pre-operational period marks the interval from the earliest beginnings of cognitive representations in the form of concrete imagery and rudimentary symbolic play to the time in which the child's conception of his environment and its operation is coherently organized.

The Concrete-operational Period

According to Piaget, children enter concrete-operational stage around the age of 7 years, on the average. The entry into this stage is the most decisive turning point in the entire course of cognitive development. The children's thinking who have attained the concrete-operational level bears a marked resemblance to the thinking of adults but the mental operations of this stage work only when they are being applied to information that the child has directly perceived. They do not work when they are being applied to information that is abstract and purely hypothetical. Since children belonging to this stage could deal only with concrete and tangible information it has been named as the concrete-operational period. Piaget believes that certain logico-mathematical structures make very good models of the actual organization and process of cognition during concrete-operational period. Thus, if Piaget says that the classificatory behaviour of the eight year old indicates that he possesses the grouping of logical class addition, he means that the child's thought organisation in the classificatory area has the properties of a Group, i.e., reversibility, identity,

associativity, composition, etc., which define this logico-algebraic structure.

Piaget (1967) claims that concrete mental operations can be grouped into two broad categories based on the kinds of information available in our environment : logico-arithmetic operations and spatial operations. Logico-arithmetic operations are contents that involve discontinuous information and spatial operations are contents that involve continuous information.

The Formal-Operational Period

The approximate age range for the onset of this stage, what we usually call adolescence, is roughly from 11 to 15 years. In most of the psychological theories of adolescence the major emphasis has always been on the emotional and social upheavals that occur during this period of life. Adolescence is usually portrayed as a period of trouble and turmoil (Ariesen, 1968). Adolescents are pictured as being in a state of constant and unrelieved conflict. They are said to be emotionally unstable and subject to bouts of depression. They are described to be torn between the desire to remain children and the need to assume the responsibilities of adulthood. Thus, the traditional view of adolescence is somewhat gloomy.

Piaget, in sharp contrast to the trouble and turmoil view, regards adolescence as the most exhilarating and productive time of life. According to him it is the time when one plans one's future and fixes the goals for life. Adolescence is a time of

great hopes and a time when simple answers to the burning questions are just not good enough. Piaget finds the thinking and reasoning of adolescents praiseworthy. He believes that intelligence reaches its peak between the age of 11 and 15 years. The thinking and reasoning during this period is clearly superior to that of childhood and it may even be superior to that of adulthood.

The reasoning at this stage is said to be hypothetico-deductive. Unlike the concrete operational thinking which operates on hard tangible facts, formal thought extends beyond the confines of everyday experience and it is not tied up with perception and memory. The formal-operational thinking involves deducing conclusions from propositions which are hypothetical rather than facts actually verified by the adolescents. Piaget has drawn an important conclusion from the features of the hypothetico-deductive reasoning, i.e., the mental operations at the formal-operational stage may be executed from start to finish at a purely symbolic level. This suggests that intelligence has moved away from 'things' towards 'ideas.' Thus formal-operational intelligence transcends reality.

Language also plays a major role in hypothetico-deductive reasoning. Piaget believes that such reasoning would be impossible if the child is not able to pose questions verbally. This question-formulating ability is supposed to rest on a new mental representations of the formal thought which are no longer restricted to extrapolations from external reality.

Flavell (1963) has described formal thought as 'A generalised orientation, sometimes explicit and sometimes implicit, towards problem-solving : an orientation towards organising data (combinatorial analysis), towards isolation and control of variables, towards the hypothetical and towards logical justification and proof.' Brainerd (1978) has described the features of formal-operational thought as (a) it is hypothetico-deductive (b) it is scientific and (c) it is reflective-abstractive.

Thus the adolescents at formal-operational level can accept assumptions for the sake of argument. They make hypotheses in propositions and try to test them. They can go beyond the tangible. At the formal thought stage, children also become conscious of their own thinking, reflecting on it to provide logical justifications of their judgements. They develop an ability to deal with a wide variety of relations such as proportionality or correlation. Thus, a remarkable qualitative change in thinking takes place when children enter into formal-operational stage, from the concrete-operational period. A comparative analysis of the characteristics of concrete-operational and formal-operational thought has been presented below to make clear the process of change of the concrete structures available at the third stage into formal structures.

An Analysis of the Characteristics of Concrete-operational and Formal-operational Thought

Concrete-operational Thought

Formal-operational Thought

- | | |
|---|---|
| <p>1. Classifications and serial ordering are employed in a step-by-step fashion, without relating all of the links into a system.</p> | <p>1. Reasoning begins with propositions and hypotheses. A system of what is hypothetically possible is structured and followed by empirical verification.</p> |
| <p>2. Well-mixed variables in an investigation cannot be separated.</p> | <p>2. Factors or variables in an experimental setting are seen as distinct and can be separated from one another.</p> |
| <p>3. Logical multiplication of factors is limited to one to one or two to two correspondence but a total n to n system is not yet available.</p> | <p>3. A complete combinatorial system is available so that all combinations of factors (n to n) can be exhaustively tested.</p> |
| <p>4. Reversibility in operations is limited to negation or reciprocity but they are not interrelated as a system.</p> | <p>4. Reversibility is advanced by the inter-relationship of negation and reciprocity, resulting in the ability to maintain a dynamic equilibrium in a system involving many factors.</p> |
| <p>5. Experimental variability is seen as a result of multiple causes and no systematic efforts are made to isolate and control factors or variables.</p> | <p>5. Variables can now be controlled systematically.</p> |

Concrete-operational
Thought

Formal-operational
Thought

- | | |
|--|---|
| 6. Chance or probability ideas are influenced by previous results, yielding a kind of gambler's fallacy. | 6. The notion of probability becomes operational and widely applicable. |
| 7. The notion of correlations is incomplete and errors occur when anything other than simple relationships are involved. | 7. Correlations are no longer limited to 1 or 1/2 but are applicable, at least qualitatively, in such situations as 1/6, 2/7, 3/11 and so on. |
| 8. Proportions are not as yet available except in the simplest sense. | 8. Proportions are now much more widely applicable in solving problems. |
| 9. Experimental contradictions cannot be pursued in a systematic manner. | 9. Because of the complete combinatorial system, experimental contradictions can now be isolated and solved. |
| 10. Conservation is limited to those physical qualities that are easily tested. | 10. Conservation is no longer limited to the immediate testable environment. |
| 11. Thinking is a derivative of the child's own actions on concrete reality. | 11. Coordination of reference systems is possible. |
| 12. 'Models' represent the concrete reality that seems to be offered rather than abstract possibilities worked out. | 12. In an experimental situation involving many variables, logical pairs are sorted out and tested. |
| | 13. The utilisation of a proof based on 'all other things being equal' is now pursued. |
| | 14. The binary system of propositional logic based on the operations of conjunction, disjunction and implication is operable. |
| | 15. The IVT group transforms the binary system into a fully operational interconnected system. |
-

To quote Piaget et al (1977), the Geneva school considers the following five characteristics as the *sine qua non* of the formal-operational stage :

1. The adolescent pupil should be in a position to state as well as test hypotheses. This type of reasoning is called the hypothetic-deductive one.
2. The adolescent pupil should be able to make the effective use of propositional logic.
3. The adolescent pupil should be in a position to separate form from content, and possibility rather than reality/^{should}become, the chief distinguishing characteristic of his thought.
4. The adolescent pupil should be able to deal effectively with the entire combinatorial nature of operations, i.e., from the 16 binary combinations to 256 tertiary operations. It is a form of closed-unit system in which passing from one element of structure to another is always possible.
5. The adolescent should be in a position to generate all the possible cases which are derivable from one single identifiable mental structure, i.e., the INCO Group where the scripts have their usual meanings: I(identity), N(negation), C(reciprocity), and CO(correlative or dual operation).

Jean Piaget (1972) has also hypothesized the existence of the fifth stage as well which he attributes to a aptitude variation, specialization and also commitment to a particular career.

JULIAN III

Adolescent Thought : A Critical
Review

CHAPTER III

Adolescent Thought : A Critical Review

Introduction

Most of the research conducted on Piaget's theory outside Geneva deals with cognitive contents belonging to the concrete-operational stage. It appears, journals that publish research on the psychological development of children receive more papers on the concrete-operational stage than on any other subject (Brainerd, 1978). There is a dearth of research studies on the formal-operational thought. In comparison to the number of excellent works published on affective, social and emotional domain of adolescents, little work has appeared in the psychological literature on the formal thought (Inhelder & Piaget, 1958). Whatever has been done, a majority of that has concentrated only on the determination of the various stages of development at which the different adolescents have been operating. Very few research studies have tried to find out the relationship of the development of formal thought to the other cognitive, cultural, social and personality traits of the adolescents. Still fewer have tried to analyse the formal thought mathematically to identify its structure comprehensively.

The studies belonging to the different areas are presented here in the tabular form for the sake of brevity and wholeness before working out their implications towards the rationale of the study at hand. They have been grouped, with respect to

the field and nature of different studies, into six categories comprising sections

- i) On stages of development.
- ii) Regarding the relationship of formal thought with age.
- iii) Related to sex differences.
- iv) On relationship between formal thought and intelligence.
- v) Regarding the relationship between formal thought and culture.
- vi) On academic achievement, personality and adjustment in relation to formal thought.

Studies on Stages of Development

Sr. No.	Name of the author(s) and year	Title of the study	Main Findings
1	2	3	4
1.	Chiappetta, L.F. and Tollerete, L. (1973).	The Effectiveness of Verbal Label Training in Aiding Second Grade Pupils to Transfer Their Classificatory Skills.	Majority of the late adolescents and adults in United States of America function at concrete-operational level.
2.	Jale, L.G. (1970).	The Growth of Systematic Thinking : Coplication and analysis of Piaget's First Chemical Experiment.	Majority of the adolescent subjects do not perform at formal-operations level.
3.	Quitt, L. (1972).	Adolescent Thinking a la Piaget : The Formal Stage.	The adults, normal adolescents and gifted adolescents show varying amounts of formal thought. The percentage varies from 25 to 60.

Contd...

1	2	3	4
4.	Ikini, S. (1968).	quantity concepts in college students.	Only 83% of the college students are clear about the conservation of volume concept. Thus they may be considered at concrete-operational level.
5.	Herrings, C. L. and White, J. L. (1971).	Sensitivity of Formal Operational thought.	Normal American adolescents do not reach the formal level of thinking at the age of sixteen.
6.	Law, L. (1974).	Formal Operational thought and the High School Science curriculum.	Upper level secondary students, except a few able ones, do not reach at the formal-operational level.
7.	Jackson, J. (1966).	The Growth of Logical Thinking in Normal and abnormal children.	Less than 50% of the 15 year old subjects attain a score representing formal-operations on the tasks presented to them.
8.	Joyce, E. K. (1977).	A Study of Formal Reasoning in elementary education Majors.	About 77% science teachers in elementary schools in the age group of 19 years and above are found operating at the formal level while about 8% are at the concrete level and the remaining 15% at the transitional level.
9.	Jurnacheck, G. A. (1975).	The Performance of Prospective Teachers on Certain Piagetian Tasks.	It has been found that 52% elementary school teachers operate at concrete-operational level and 42% at the formal-operational level.

Contd...

1	2	3	4
10. Tarplus, J. et al. (1973)	Intellectual Development Beyond Elementary School IV Ratio: The Influence of Cognitive Style.	One-fifth of pupils during adolescence develop firm proportional reasoning.	
11. Tarplus, J. and Trons, J.B. (1976).	Implication of Accumulating Data on Levels of Intellectual Development.	Over a very wide age range, 13-45 years, about one-third of the subjects attain formal level.	
12. Tarplus, J. and Tarplus, J. (1976)	Intellectual Development Beyond Elementary School: Deductive Logic.	Less than fifty percent of the physics teachers use formal thought during problem solving.	
13. Tusting, M.P. (1975).	Preocious Cognitive Development at the Level of Formal-operations.	Majority of the adolescents fail to perform at formal-operational level.	
14. Tohlberg, L. and Gilligan, J. (1971).	The Adolescent as a Philosopher - The Discovery of the Self in a Post Conventional World.	All normal children attain the concrete-operational level during adolescence but most of them do not attain the formal-operational level.	
15. Lawson, J.E. and Blake, A.J.M. (1976).	Concrete and Formal Thinking Abilities in High School Biology Students as Measured by Three Separate Instruments.	It has been found that 47% of high school biology students operate at concrete-operational level and 53% at the formal-operational level.	

Contd.

1	2	3	4
16.	Lawson, L. L. and Renner, J. J. (1973).	Relationship of Science Subject Matter and Develop- mental Levels of Learners.	About two-third of the adolescent pupils fail to show formal thought in their mastery over the abstract concepts.
17.	Lawson, L. L. and Renner, J. J. (1974).	A quantitative analysis of responses to Piagetian tasks and its impli- cations for curriculum.	About 22% of the college freshmen operate at formal-operational level while 51% and 27% are found at the concrete- operational and post- concrete-operational level respectively.
18.	Lee, L. L. (1971).	The Concomitant Development of Cog- nitive and Moral Modes of Thought : A Test of Selected Predictions of Piaget's Theory.	It has been found that less than 50% of the subjects operate at the final sub-stage of formal-operations.
19.	Lovell, T. and Gutterworth, J. J. (1966).	Abilities Underlying the Understanding of Proportionality.	A majority of the adolescent subjects do not perform at formal- operational level.
20.	Lovell, T. (1961).	A Follow-up Study of Inhelder and Piaget : The Growth of Logical Thinking.	Formal thought has not been attained by pupils of low academic ability even at fifteen years of age.
21.	McKinnon, J. J. and Renner, J. J. (1971).	Are Colleges Con- cerned with Inte- lectual Develop- ment?	About 50% of college freshmen operate at the concrete-operational level, 25% at the post- concrete stage and only 25% at the formal- operational level.

Contd..

1	2	3	4
+ 22. Mealinge, H.J. (1961).	Some Aspects of Problem Solving in Science.	Pupils of low academic ability even at fifteen years do not attain formal thought.	
X 23. Hacks, G. and Hacks, V. (1971).	The Development of Formal Thought as shown by Explana- tions of the Oscilla- tion of a Pendulum: A Replication Study.	Formal thought is confirmed as well as used to reject irrele- vant variables on simple pendulum prob- lems.	
X 24. Hartono, S.C. (1977).	A Developmental anal- ysis of Performance on Piaget's Formal Operations Tasks.	Even the oldest age group (18th graders) does not consistently show formal-operational performance across all tasks.	
7 25. Wordland, et al. (1974).	A Study of levels of Concrete and Formal Reasoning Ability in Junior and Senior High School Science Students.	About 86 % of the senior high school science students are at the concrete level while 14% are at the formal level.	
+ 26. Renner, J. J. and Stafford, D. C. (1973).	Teaching Science in the Secondary School.	Among the 200 students of grades ten, eleven and twelve about 66% are at the concrete level, 20% at the transitional stage and 14% at the formal-operational stage.	
X 27. Ross, R.J. (1973).	Some Empirical Parameters of Formal Thinking.	The Percentage of under- graduates operating at the formal level is above the 50% figure.	

Contd...

1	2	3	4
28. Gelwabel, H. (1973).	Formal Operations in College Freshmen.	Among the college freshmen only 20% perform at the late formal level on all the three tasks used.	
29. Bhaskar, R. (1970).	A Study of Intellect- tual Development and its Relationship with Intelligence and Achievement of 10th Grade Science Pupils.	Most of the pupils up to the age of 15 years do not attain formal level.	
30. Vaidya, H. (1964).	A Study of Problem Solving in Science Among Certain Groups of Adolescent Children.	Though adolescent pupils are in a position to state hypotheses, most of them are not in a position to test them. They do not exhaust all possibilities.	
31. Walker, R. L. et al. (1970).	Written Piagetian Task Instruments: Its Development and Use.	Formal thinking develops little even up to the age of fifteen.	

Studies Regarding the Relationship of Formal Thought with Age.

1. Clayton, V. and Overton, E. P. (1976).	Concrete and Formal Thought Processes in Young Adults and Old Age.	Age related performance differences occur on formal-operational thought.
2. Annals, P. (1967).	A Study of the Development in Logical Judgements in Science of Successful and Un- successful Problem Solvers in Grades Four Through Nine.	Formal thinking increases with age (10, 11, and grade), it proceeds in stages which are dependent upon each other.

Contd...

1	2	3	4
3.	Inhelder, P. and Piaget, J. (1958).	The Growth of Logical Thinking from Childhood to Adolescence.	As adolescence tends to approach adulthood, pupils are in a position to organize as well as mathematise their solutions.
4.	Jangel, L. and Buell, J. (1972).	Exclusion of Irrelevant Factors (The Pendulum Problem).	Formal thought increases gradually with age regardless of sex.
5.	Lin, M. and Levine, J. (1976).	Adolescent Reasoning: The development of the ability to Control Variables.	Older adolescent pupils succeed to exclude variables even when different problems are used.
6.	Portoreano, L. (1977).	A developmental analysis of Performance on Piaget's Formal Operations Tasks.	Mean scores on Piagetian tasks show an increasing trend with age.
7.	Sayre, T. and Daniel, G. (1973).	Piagetian Cognitive Development and Achievement in Science.	Formal thinking grows gradually during adolescence.
8.	Shayer, M. and Dylan, L. (1978).	The Distribution of Piagetian Stages of Thinking in British Middle and Secondary School Children 14 to 16 Year Old.	There is no increase in proportion of pupils showing formal thinking beyond the age of 15 years.
9.	Tomerville, G. C. (1974).	The Pendulum Problem: Patterns of Performance Unifying Developmental Stages.	Over all level of performance on the pendulum problem is strongly related to age, but not to sex or to the school attended.

(Contd. ...)

1	2	3	4
10. Vaidya, B. (1970).	The Growth of Logical Thinking in Science During Adolescence.	The mean performance on the various schemes of thought shows an increasing trend with grade.	
11. Walker, H.S. et al. (1970).	Written Pigeon Task Instrument : Its Development and Use.	Formal-operations are independent of age between 13 to 17 years.	
12. Weeks, J. (1973).	The relationship of Grade, Sex, Socio-economic Status, Scholastic Aptitude and School Achievement to Formal Operations Attainment in a Group of Junior High School Students.	The incompleteness of formal operations abilities at the ninth grade level indicates its subsequent growth still at higher age levels.	
13. Zozny, G.L. and Cox, J.L. (1975).	The Effects of Sex Differences on the Assessment of Formal Operational Thinking.	It has been found that ability to separate variables increases with age.	
14. Tulin, E.H. (1966).	Formal Thought in Adolescence as a Function of Intelligence.	Formal thinking increases with age.	

Studies Related To Sex Differences

1. Graybill, J.L. (1974).	A Study of Sex Differences in the Transition from Concrete to Formal Thinking Patterns.	It has been found that sex differences favouring boys begin to appear at the age of eleven.
------------------------------	---	---

Contd...

1	2	3	4
2.	Graybill, L.A. (1975).	Sex Differences in Problem Solving Ability.	Boys begin to score at formal level at 13 yrs. while girls lag-behind.
3.	Lawson, A.L. (1975).	Sex Differences in Concrete and Formal Reasoning Ability as Measured by Manipulative Tasks and Written Tasks.	Sex differences favouring males in formal reasoning are noticed.
4.	Rajput, H.D. (1975).	A Study of the Scheme of Proportion among certain Groups of Adolescent Pupils.	No significant sex differences exist on the scheme of proportion.
5.	Thayer, H. and Lynn, J. (1978).	The Distribution of Abstract Schemes of Thinking on Proportion middle and secondary school children 14 to 16 Year Old.	The simple pendulum problem shows no sex differences.
6.	Conervilla, C.L. (1974).	The Pendulum Problems Patterns of Performance Refining Developmental Stages.	Formal thought hardly depends on sex and the type of schools.
7.	Weeks, T.L. (1973)	The Relationship of Grade, Sex, Socio-economic Status, Scholastic Aptitude and School Achievement to Formal Operations Attainment in a Group of Junior High School Students.	Significant sex differences are found on conservation of volume and other aspects.

Studies on Relationship Between Formal Thought and Intelligence

1	2	3	4
1. Case, R., and Collinson, J.R. (1962).	The Development of Formal Thinking in Verbal Comprehension.	Children of matching CA and MA had different scores on formal thought. Since some other factors such as cultural back- ground, range of expe- rience and verbal reper- toire may be contributing to the development of formal thought.	
2. Clayton, V. and Overton, E.G. (1976).	Concrete and Formal Thought Processes in Young Adulthood and Old Age.	Except for the young sample, the operational tasks were found to be unrelated to fluid intelligence.	
3. Cloutier, R. and Goldschmid, R.S. (1976).	Individual Differ- ences in the Development of Formal Reasoning.	Significant correlations have been obtained between scores on the proportion test and non-verbal inte- lectual capacity as measured by the Raven's CPM.	
4. Khan, D. (1976).	Relation of two Piagetian Stage Transitions of IQ.	A high correlation has been found between mental age and progression towards Piaget's stage of concrete-operations. However, the correlation is non-significant be- tween mental age and prog- ression towards Piaget's stage of formal-opera- tions.	

Contd...

1	2	3	4
5. Vaidya, N. (1964).	A Study of Problem Solving in Science Among Certain Groups of Adolescent Children.	A given problem is solved over a wide IQ range.	
6. Stephens, L.L. et al.(1960).	The Development of Reasoning, Moral Judgment and Moral Conduct in Adolescents and Normals.	Significant correlations of WISC verbal IQ, performance IQ and Hull Scale IQ with Piagetian tasks of reasoning and formal operations have been reported for subjects 6 to 12 years.	
7. Valentine, J.L. (1975).	Performance on Two Reasoning Tests in Relation to Intelligence, Divergence and Interference Proneness.	The results show that convergent intelligence is a necessary but not a sufficient condition for success on the tasks.	

Studies Regarding the Relationship Between
Formal Thought and Culture

1. Jaita, J.B. (1975).	A Study Comparing College Science Students' Performance on Piagetian Type Tasks, Including Cross-Cultural Comparisons.	No significant differences are found between cultural background and over all performance on Piaget type tasks among college science students.
2. Henry, G.O. (1974).	The Effects of Culture and Education on the Acquisition of Formal Operational Thinking.	It has been found that a suburban culture background promoted the development of formal operations.

contd. next

Studies on Academic Achievement, Personality
and Adjustment in Relation to Formal Thought.

1	2	3	4
1. Blasi, A. and Hooffel, J.D. (1974).	Adolescence and Formal Operations.	It has been concluded that personality develop- ment during adolescence may take place independ- ently of formal opera- tions.	
2. German, J.G. et al.(1976).	The personality of the child and the utilization of operative thought.	Significant correlations between different measures of operational thinking and different personality variables have been obtained.	
3. Tathum, J.L. (1975).	The Unique Contri- bution of Piagetian Measurement to Diag- nosis, Prognosis, and Research of Children's Mental Development.	It has been found that Piagetian factors are having a dominant con- current association with the measures of school achievement thus point- ing out the way to new and possibly more reli- able and valid predictors of achievement.	
4. Osicki, K.J. (1973).	Affective and Cognitive Develop- ment: Comparison of High Achievement and Risk Level with Pia- getian Levels of Cog- nitive Development for Two Socio-eco- nomic Groups.	Cognitive development does not vary with either n.ach. or risk level.	
5. Sayre, B. and Daniel, J.B. (1975)	Piagetian Cognitive Development and Achievement in Science.	No significant relation- ship has been found be- tween the scholastic achievement and level of formal operations.	
6. Vaidya, M. (1979).	The Growth of Logical Thinking in Science During Adolescence.	The top group of success- ful problem solvers differs significantly from the bottom group in home adjustment, health adjustment and school adjustment.	

Concluding Statement

Review of research studies presented on the preceding pages under the heading 'Studies on Stages of Development' shows that the percentages of adolescents (11 to 15 years) operating at formal-operational level vary on quite a wide range (approx. 15% to 60%) in different countries and cultures. There are some interesting studies in which it has been found that even the college students (above 15 years) and elementary as well as other learners operate in good numbers only at concrete-operational level. Though the studies reviewed do not cover the whole research conducted on this topic, yet if they are taken as a representative sample then the general trend emerges that the adolescents do not attain the level of formal thought upto the age of 15 years. That is why Piaget (1972) has started thinking about the existence of a fifth stage which may extend the period of growth to 20 years of age which was earlier considered upto 15 years of age. Thus, when it has been found that the adolescents do not operate fully at the formal level then it becomes very clear that they operate (mostly) at the semi-concrete and semi-formal operational level. Therefore, it becomes very necessary for a research worker, first of all, to determine the structure of this complex mode of thinking (adolescent thought) so that the effects of the different variables related to the nature of the determined factors of adolescent thought may be studied later on in depth as Royce

(1950) has pointed out that a proper order of research program might be ; first, to use a set of a priori measures in a field of investigation and factor analysis then to determine the basic traits or the other sources of variance operating; second, to study these factors, one at a time, by the technique of analysis of variance to determine how they are affected by the different experimental conditions or how they vary among groups that differ with respect to age, sex, education or the other pertinent background variables; and lastly, to study them experimentally in the laboratory for specific groups under carefully controlled conditions.

"The studies regarding the relationship of 'formal' thought with 'age' show that formal thought increases with age. To instead of taking a static picture at one age level, it seems more desirable to study formal thought at different age levels during the formal-operational stage (11 to 15 years). 'Studies Related to 'Sex Differences' are not unanimous in establishing whether sex differences exist or not with respect to the development of formal thought. Thus, it needs further verification. Similar is the position of the variables of intelligence, academic achievement, adjustment, personality and culture, etc.

Status of Research on Adolescent Thought

The remarks of Inhelder & Piaget (1958) regarding the status of research in the area of adolescent thinking are worth noting as they pointed out "It is surprising that inspite of the large number of excellent works which have been published on

the affective and social life of the adolescent - we hardly need remind the reader of the studies of Stanley Hall, Jaspars, Montousse, Pranger, Charlotte, Juhler, Laidis, Lavinia Lewis, Brodis, Fleming, or Debesse, or those by psychoanalysts such as Anna Freud and Helene Deutsch, and by sociologists and anthropologists such as Maclintock and Margaret Mead, nor to mention others - so little work has appeared on the adolescent's thinking." Whatever scattered researches have been undertaken on adolescents in Mexico and Latin most of them have simply tried to identify the stages of development of the subjects under study. A few studies have attempted to investigate the relationship of the adolescent thought with the other outside variables. Still fewer could manage to determine the structure of the adolescent thought and that too only with respect to one or two dimensions of it. Perhaps, none has tried to analyse the comprehensive contents of the adolescent thought and its various dimensions to identify the global structure of adolescent thought.

In India the situation is still worse. Vaidya (1975) reported : "It is a research desert characterized by the lack of personnel, problems and publicity". Some research workers have started grappling with the problems related to the adolescent thinking.

Problems Posed in the Field

Piaget and his co-workers have investigated the area of cognition without considering or giving much importance to the other variables like intelligence, personality traits, socio-

economic status, etc. It is very necessary to explore the area further with respect to as many independent variables as possible. It does not mean that the past adventures on thinking have all been fruitless since on their basis we have known the road yet to travel.

Still today very little is known on thinking in relation to the past history of the individual. Thus, 'what makes people attack problems' according to Cohen (1964) may contribute to our understanding of thinking processes. Vincke (1962) has mentioned that the whole area awaits invasion through case study approach with a view to collect as well as to interrelate as many aspects of performance as possible in as many situations as practicable. Duncan, Hecker, 1961, Trowers and Watson have also emphasized the importance of investigating thinking in relation to some outside variables like intelligence, personality traits, socio-economic status, motivation, ego involvement, etc. (Vaidya, 1975). The stage concept propounded by Piaget also needs to be investigated in depth alongwith the emergence of various mental operations at various age levels. Lovell (1973) while carrying out several studies on developmental processes in thought among children varying widely in age, intelligence and culture, has suggested the following problems which cry for solutions:

1. What is the role of experience (physical and mathematical) in intellectual development of children? How is it to be handled?

2. What had Start School in 1970, what is the long term influence or impact of early stimulation on the culturally deprived and on certain types of school educable retarded children?
3. What is the effect of variables like emotional life, teaching learning techniques based on Piaget's works, culture and sub-culture patterns and the restricted functioning of any schema within a given area of knowledge at one time on cognitive growth?

There are several difficulties which need to be tackled or mastered before we can understand clearly the nature of thinking : problem solving, concept development and attainment. The major difficulty lies in our failure to understand the sequence of reasoning from the very early childhood to late adolescence, not only within each age-group but also across the various age groups over a very wide range. Precisely speaking, the field of human thinking as a whole poses many fundamental problems which are yet to be investigated, even partially, before we can fully understand it. The most crucial problem of human thought, more accurately adolescent thought, is the understanding, identification and determination of its underlying mathematical structure which the present study has attempted to tackle.

Distinguishing Characteristics of the Present Study

This study takes its inspiration from the Geneva school. Jean Piaget, being its leading advocate, has contri-

buted to the whole field of psychology immensely for the last sixty years or so. The present study has some distinguishing characteristics over the studies of Piagetian context as follows:

1. It attempts to psychometrize the test of Piaget "Type" tasks, which can be administered in the group setting.
2. It draws a large size of sample for obtaining meaningful results.
3. It includes a large number of outside variables with a view to investigate the phenomenon in depth.
4. It subjects data, so collected, to a highly mathematical technique - factor analysis - rarely used by the earlier research workers.
5. The tasks, included in the test of Piaget "Type" Tasks, involve a continuous chain of reasoning.

VI

and procedure

CHAPTER IV

Plan and Procedure

Section A

Frame Work of the Study

Introduction

As stated earlier this study was undertaken to determine the mathematical measure of adolescent thought (described in Chapter II) through factor analysis. The usual technique of factor analysis starts with a correlation matrix of a set of variables which is reduced to comparatively very small number of factors to explain the underlying nature and behaviour of the variables. On the basis of the results obtained through factor analysis a theory or a mathematical model can be formulated and vice versa, i.e., a model or a theory if postulated beforehand is verified. It was in line with the former approach that the study was launched to identify a mathematical model of adolescent thought. In addition to this, it was intended to seek evidence regarding the relationship of some independent variables such as age, sex, intelligence, reasoning ability, space relations, academic achievement, adjustment and other personality traits with the different dimensions of adolescent thought measured by the Piaget Type Tasks. Specifically speaking, the investigation was guided by the objectives given below:

- a) To develop a reliable and valid paper-pencil test of Piaget "type" tasks to procure data regarding the various dimensions of adolescent thought.
- b) To determine the relationship between the performance on Piaget "type" tasks and the independent measures of reasoning, intelligence, reasoning ability, space relations, academic achievement, adjustment and other personality traits (H₂).
- c) To find out the effect of age and sex on the performance on Piaget "type" tasks.
- d) To identify the factorial structure of adolescent thought.

Hypotheses

The following hypotheses were proposed to be tested through this study:

1. Does the performance on Piaget "type" tasks increase with age during the formal-operational period?
2. Whether boys and girls perform equally well on Piaget "type" tasks?
3. The measures of intelligence, both verbal and non-verbal, correlate significantly with the measures of the dimensions of adolescent thought.
4. There exists a significant relationship between the measures of academic achievement and the measures of the dimensions of adolescent thought.
5. The measures of reasoning ability and space relations yield a significant correlation with the various measures of adolescent thought.

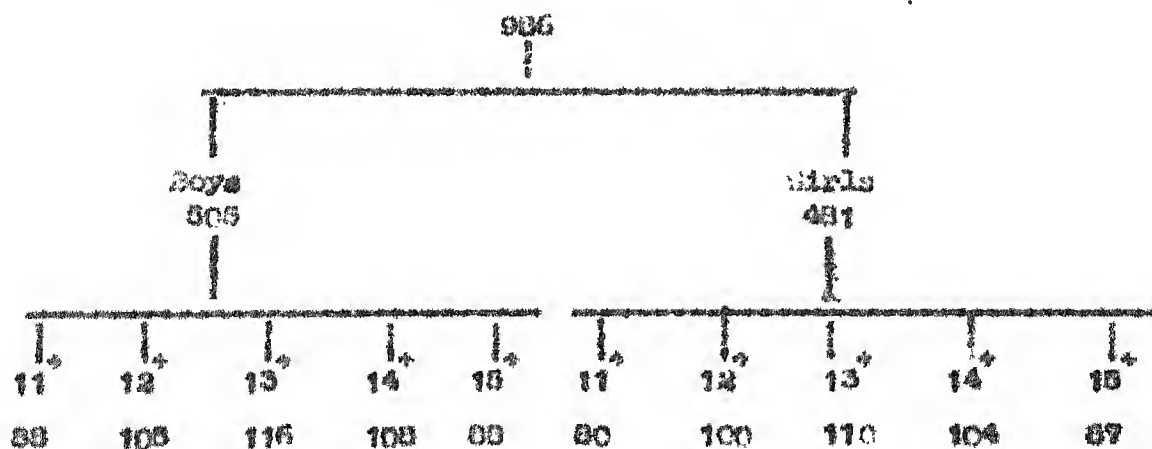
6. The measure of adjustment is significantly related to the performance on Piaget "type" tasks.
7. The measures of personality (15%) exhibit significant relationship with the measures of the dimensions of adolescent thought.
8. The performances on Piaget "type" tasks form an inter-related measure of adolescent thought and exhibit a unifactor structure.
9. The measures of intelligence, academic achievement, reasoning ability, space relations, adjustment and personality cluster in specific constellations with the measures of dimensions of adolescent thought explaining thereby the common factor variance.

Sample

The sample for the study was drawn among the students of twelve high schools situated in the rural areas of Multan and Faisalabad districts of Punjab and administered by the Department of Education, Govt. of Punjab. A purely rural sample was taken, first, to avoid heterogeneity which would have been caused by pooling together the urban and rural samples as the research findings (Jacilia, 1973) show. Second, the schools in the rural areas of Punjab are having a homogeneous population of students as they possess a similar socio-cultural background. Third, the academic atmosphere and the schooling facilities provided by the government schools in the rural areas are almost identical. It may be recalled that the study was planned to determine the mathematical structure of formal thought. The expected age range for the development of formal thought is 11 to 15 years.

To draw a representative and homogeneous sample of 11 to 15 year old students (numbering about 3000) studying in the above mentioned twelve high schools, the dates of birth of all the students were noted down. And to control age and grade simultaneously only the students at 11⁺, 12⁺, 13⁺, 14⁺ and 15⁺ year age levels and studying in VI, VII, VIII, IX and X grades respectively were picked up. Then a random sample of about 100 students was drawn from each school, taking almost equal number of boys and girls, from each grade. The final sample consisted of 986 students (505 boys and 481 girls) who appeared in all the tests and whose other records were also complete. The frame of the final sample was as given below:

Total sample



Data Collection Schedule

It took three days to take data on a group of 80 students from ^a school. On the first day the classwise lists of the students studying in the school were taken. The dates of birth

were noted down to draw a representative sample from each age or grade level. The academic achievement scores of the students included in the sample were taken for the five school subjects, i.e., Mathematics, Science, English, Punjabi and Hindi. Four tests were administered on the second day -- Rathana's Adjustment Inventory and Cattell's IPI in the morning session and Binet Type Tasks (Part I) and Cattell's Culture Fair Intelligence Test in the evening session. Again four tests were administered on the third day. Binet Type Task (Part II) and Jolota's GVI in the morning session and Space Relations (1973) and Abbey's Reasoning Ability Test in the evening session. Thus it took 6 complete days to gather data on the sample of one school. In all, data were collected from twelve schools. The school teachers' help was of immense importance in the process of administration of tests and procuring of records such as dates of birth and academic achievement scores, otherwise, the task would have been very cumbersome for the investigator to accomplish.

Statistical Treatment of the Data

There are two major schools of thought regarding the operational aspect of factor analysis. The first may be named as the British school in which reference could be made to the works of Spearman, Burt, Brown, Thomson, Stephenson, Aikens, Aycock, Blizinger, Brown, Cattell and Vernon. They advocate Hierarchical Group Factor Theory. The second is the American school of thought where the works of Thurstone, Kelley, Paterson and Elliot, Alexander and Guilford are worth noting. They believe in Multiple Factor Theory. According to the first school

of thought "all branches of intellectual activity have in common one fundamental function (or group of functions) whereas the remaining or specific elements of the activity are seen in every case to be wholly different from that in all others." The second school of thought postulates that the cognitive functions are based on "a number of" components of more nearly equal variance" the multiple factors. Though both the schools have their own specifications regarding the interpretations of the common variances operating among the different tests of cognitive abilities, yet they convey the same sort of information with a little variation in the language used for explanations. Blasinger and Hyman (1933) and Jensen (1939) demonstrated quite early that the contents of group factors correspond very closely to that of multiple factors. This study took inspiration from the first school of thought, hence, the technique chosen for factor analysis was that of Principal Axis Method.

The data regarding the 34 measures were put into 34 x 34 correlation matrix and subjected to factor analysis to determine the factorial structure of adolescent thought. The computations were carried out through "ELIAC - 1022 Computer" at Computronics India, New Delhi using 24-1 programs for factor analysis given in Statistical Package for the Social Sciences (SPSS) by Nie, et al. (1970). One-way analysis of variance technique was used to determine the age and sex differences regarding the performance on Piaget Type tasks at different age levels. The relationships between the measures of the dimensions of adolescent thought and the measures of the independent variables,

namely, intelligence, reasoning ability, space relations, academic achievement, adjustment and personality were worked out through bivariate analysis. The descriptive statistics were also computed to know the distributions of the various measures included in the study.

Section B

Description of the Test of Piaget Type Tasks

Being a factor analytical study which requires a sufficiently large sample, particularly, when the number of variables handled is large, the data were to be collected in group settings. A paper-pencil test for group administration, consisting of Piaget Type Tasks, was developed by the investigator which in the final form consisted of ten tasks - five each in Part I and Part II.* These tasks were the simplified forms of the ones already used by various research workers as an interview technique or for the individual administration in different investigations of formal thought. The following dimensions of adolescent thought were covered by these tasks

Part I

Task No.	Dimension of Adolescent Thought Covered
1	Classification
2	Grouping of Thought
3	Application to Arithmetical and Spatial Problems.

* See appendices (i) and (ii)

<u>Task No.</u>	<u>Dimension of Adolescent Thought Covered</u>
4	Permutations and Combinations
5	Ratio and Proportion

Part II

6	Formulation of Probing Questions
7	Interpretation and Coordination of Information
8	Stating and Testing Hypotheses
9	Space Visualization
10	Grasping the Essence of the Problem

The objective of each task alongwith the detailed description and mode of administration have been presented below:

TASK NO. 1

Objective

The task was designed to serve as a measure of classificatory ability. The purpose of the task was to find out if the subjects can work out a criterion on the basis of the common properties of the objects for their classification. Both primary as well as secondary classification abilities are expected to be developed at the formal-operational stage.

Description

This task consisted of 27 geometrical figures which could be classified into three categories with respect to shape (9 circles, 9 triangles and 9 rectangles) or size (9 small, 9 medium and 9 large) or colour (9 white, 9 shaded and 9 crossed). In every case, each category could be further subjected to

secondary classification making three sub-categories on any two of the three criteria (shape, size and colour) leaving aside the one on which the primary classification has been done.

The figures were printed on a sheet of paper, keeping in view the maximum possible combinations of different figures with respect to the classificatory criteria (shape, size and colour) so that the subjects have to make equal efforts to classify the figures on any of the criteria mentioned above. The subjects were asked to do first primary classification and then secondary classification selecting a criterion of their choice in each case. It was for the subjects to identify the criteria mentioned earlier. Scoring was done awarding one score for each correctly classified figure. The maximum score on this task could be 54 and the minimum as zero.

Administration

A gardener gave a bunch of flowers of different types to his son and asked him to classify the flowers into different groups. The boy put the roses in one group, the chameli in the other and so on. Then the gardener asked him to further sub-classify the roses into different categories. The boy put the red roses in one category, the white roses in the other and so on. Now you go through the task given ahead and try to perform as demanded.

There are given some geometrical figures on the opposite page (Fig.1). Each figure has been assigned a number for its identification. Classify these figures into three main categories in such a way that similar figures come under one category.

You are to write only the numbers of the figures under the categories, I, II and III to which they belong.

Category I

Category II

Category III

Now, further sub-classify each category into three sub-categories in such a way that similar figures of each category come under the same sub-categories of the same.

Category I
Sub-categories
A B C

Category II
Sub-categories
A B C

Category III
Sub-categories
A B C

TABLE NO. 3

Objective

The task was meant for the determination of the development of various postulates of grouping of thought; such as, closure, associativity, identity, inverses and tautology*. The combination of the development of all these postulates has been referred to as the measure of grouping of thought.

Description

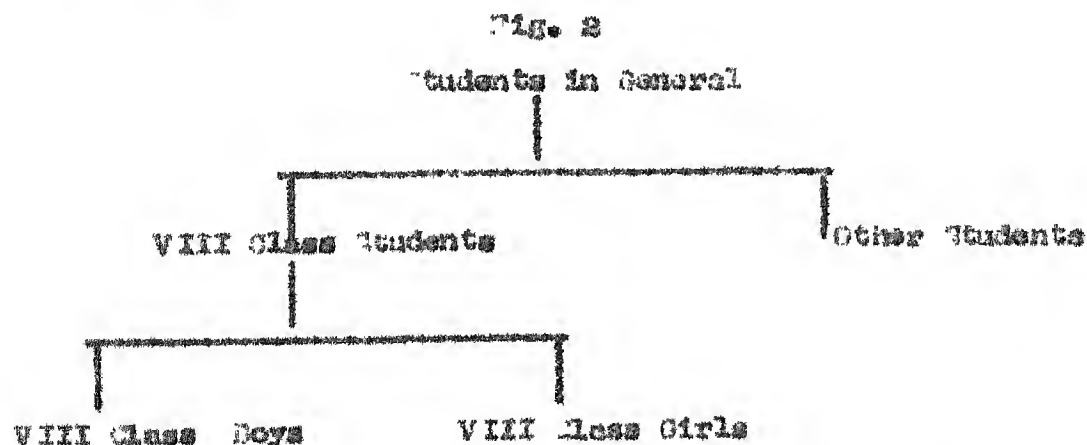
In this task a classification hierarchy printed on a paper was provided to the subjects. The classification hierarchy

* For further details see appendix (iii)

showed that the students in general had been classified into two categories, i.e., VIII class students and other students and then VIII class students were further classified into two categories of VIII class boys and VIII class girls. The subjects were asked to answer seven questions framed regarding the five postulates of the grouping of thought, i.e., closure, associativity, identity, inverse and tautology. One score was awarded for each correct answer, thus, making the maximum score on this task to be 7 and the minimum to be zero.

Administration

Look through the classification hierarchy given below in Fig. 2 and with the understanding of it answer the questions given above.



1. If VIII class boys and VIII class girls are grouped together, what name will you give to the group formed?
2. If VIII class boys are grouped with VIII class students and then both in combination are grouped with students in general, what name will you give to the group formed?

3. If VIII class students are grouped with students in general and then both in combination are grouped with VIII class boys, what name will you give to the group formed?
4. If VIII class boys are grouped with a class without students what name will you give to the group formed?
5. If VIII class boys are grouped with VIII class girls and then VIII class boys are taken out from the group, what name will you give to the group left behind?
6. If VIII class boys are grouped with VIII class boys, what name will you give to the group formed?
7. If VIII class boys are grouped with VIII class students, what name will you give to the group formed?

TASK NO. 3

Objective

The assessment of the ability to generalize was the purpose of this task. Generalization of both the arithmetical and algebraic series with respect to the operations of addition, subtraction and multiplication had been taken into account. The overall score represented the measure of generalization to arithmetical and algebraic symbols.

Description

This task consisted of three series of numbers based on the mathematical operations of addition, subtraction and multiplication in the beginning and later on they were symbolized

to algebraic series under the same mathematical operations as mentioned above. Some spaces were left blank towards the end of arithmetical as well as algebraic series so that the subjects may understand and internalize the logic behind the formulation of these series in the beginning and then on the basis of the generalization they may fill in the blank spaces. The scoring was done allotting one score for each correct entry in the blank spaces. The maximum score on this task may go upto 30 and the minimum to zero.

Administration

Three sets of numbers and algebraic symbols having two columns each are given below. In each set you will find some relationship between the entities of the two columns. Try to understand the relationship and fill in the blanks given in each set.

1	2	3
2	5	-7
6	9	-13
10	13	-19
14	(.....)	-25
(.....)	(.....)	(.....)
5 x 2	=	10
5 x 3	=	15
5 x (...)	=	20
(....) x 5	=	25
(....) x (....)	=	(.....)
1	1+3	-1
1 + 4	1+7	-1+6
1 + 8	(.....)	-1+12
(.....)	(.....)	(.....)
5 x 1	=	5
5 x (1 + 1)	=	5(1 + 1)
5 x (1 + 2)	=	(.....)
(.....) x (.....)	=	(.....)
(.....)	3	(.....)
(.....)	(.....)	(.....)
5 x (.....)	=	3
(....) x (....)	=	(.....)

PART IV. 4

Objective

The task was designed to assess the ability of the subjects to make the various permutations and combinations of the given materials and the operations. Thus, the performance on the task was taken as the measure of the ability to make permutations and combinations.

Description

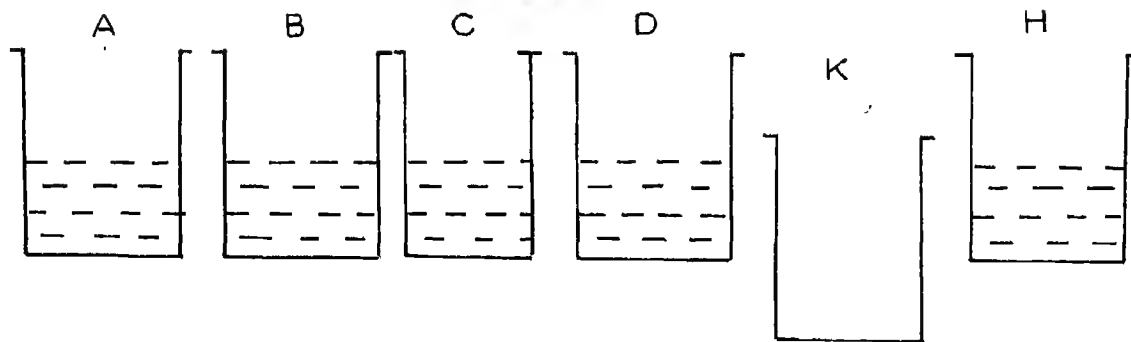
The task consisted of four beakers (1, 2, 3 and 4) containing colourless chemicals and a fifth beaker (5) which contained an indicator but that too colourless. There was another empty beaker (6). The subjects were to make the imaginative permutations and combinations as all the above beakers were presented to the subjects just as diagrams on the paper.

The problem was posed as that a student while experimenting with these chemicals suddenly got a yellow colour when he mixed some of the chemicals from the beakers 1, 2, 3 and then put the small quantity from the beaker 5. What possible tries (permutations and combinations) they could make to get the same colour again. Thus the subjects could make the combinations taking one, two, three and four beakers at a time. One score was awarded for each un-repeated correct combination. The highest score on this task could be 13 excluding examples and the minimum being zero.

Administration

Four beakers A, B, C and D are placed on a table (Fig. 3). The fifth beaker E is also placed a bit away. All the beakers are filled up with different colourless chemical reagents. There is one empty beaker K. You performed an experiment with these chemicals one day. You took out the chemical reagent from some of the A, B, C and D beakers and put into the beaker K. Then you took out the reagent from beaker E and put it also into the beaker K. Thus, the contents of the beaker K became yellow in colour. What experiments will you perform to find out the reagents which on putting together had made the yellow colour appear? Describe all possible experiments.

Fig. 3



Examples:

Experiment No. 1 Took out the chemical reagent from the beaker A and put it into the empty beaker K. Then took out the chemical reagent from the beaker E and put it also into the beaker K.

Experiment No. 2 Took out the chemical reagents from the beakers B and C and put them into the empty beaker K. Then took out the chemical reagent from the beaker E and put it also into the beaker K.

Similarly, you write down the other possible experiments.

TASK III. 5

Objective

The objective of the task was to assess the development of the concept of ratio and proportion among the adolescents. The simple whole number ratio and the complex ratio operations had been dealt with to get the measure of the ratio and proportion.

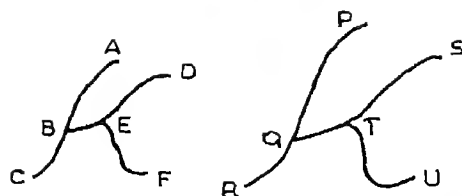
Description

A letter 'K' in two different sizes was printed on a paper. The dimensions of the small size and the large size of the letter were in the ratio of 2:3. The lengths of the arms of the small size letter were given and the students were to find out the corresponding lengths of the arms of the large size letter. One score was awarded for each correct answer where the lengths of the arms were the simple multiples of the given ratio and two scores for each correct answer were awarded where complex ratio operations were involved. The maximum score on this task could be 7 and the minimum as zero.

Administration

A letter 'K' is given below (Fig. 4) in two different sizes - one small and the other large. It is given that the lengths of the arms of the small letter and the large letters are in the ratio of 2:3. On the basis of this information answer the questions given below:

Fig. 4



1. If length of the arm AE is 4 cms., what will be the length of the arm DE ?
2. If length of the arm PT is 8 cms., what will be the length of the arm ST ?
3. If length of the arm BC is 16 cms., what will be the length of the arm BE ?
4. If length of the arm BC is 10 cms., what will be the length of the arm BE ?
5. If length of the arm ST is 9 cms., what will be the length of the arm PT ?

TASK NO. 5

Objective

The purpose of the task was to determine the extent of the development of the ability to formulate probing questions about any object or its functioning which is a basic characteristic of the formal thought. The over-all performance was designated as the measure of the formulation of probing questions.

Description

The task was meant for obtaining the measure of the ability to formulate probing questions. The subjects were to

ask questions quite novel in nature, about the bicycle and the cow, whose answers were beyond their comprehension. Thus the subjects were to probe into, imaginatively, all types of situations wherever they find something un-understandable or unknown about the above mentioned objects. It was an open ended task. One score was awarded for each correct (meaningful) question posed. The maximum score obtained on this task was 24 and the minimum was zero.

Administration

The students of your age are very curious to know about the things in their environment. Number of questions come to their minds whose answers they do not know. For example, Mohan, a student of your age asked the following questions about the sun:

1. Is the sun a ball of fire in reality?
2. Why the sun does not fall on the earth?
3. Can we live without the sun?
4. What is the temperature of the sun?

Thus many questions might have been coming to your mind also.

You please write down as many questions as you can, whose answers you do not know, about (a) Bicycle and (b) Cow.

(a) Bicycle

(b) Cow

TABLE No. 7

Objective

The task was designed to assess the ability to interpret and coordinate a given information. Thus, the performance on the task represents the measure of the ability of interpretation and coordination of a given information.

Description

This task consisted of a board having nine squares printed on a paper. The subjects were informed that first these squares were painted row-wise with blue, white and red colours respectively and then these squares were painted column-wise with red, blue and white colours respectively. This new colours emerged on those squares which got paints of two different colours and the others remained of the same colour which got the paints of the same colour twice. The subjects were to interpret and coordinate the information to find out the colours of which the different squares appear, when both the painting strategies were over. Scoring was done awarding one score for each correct answer. The maximum score on this task could be 9 and the minimum as zero.

Administration

Here is given a board having traced nine squares upon it (Fig. 8). The top three squares (A, B, C) were painted blue, the middle three squares (D, E, F) were painted white and the bottom three squares (G, H, I) were painted red. When they were painted second time it so happened that the left three squares (A, D, G) got painted red, the middle three squares (B, E, H) got

painted blue and the right three squares (4, 5, 6) got painted white. Thus the different colours got mixed and changed in the various squares as follows:

1. The square which was painted blue and red or vice versa looked gray in colour.
2. The square which was painted blue and white or vice versa looked light blue in colour.
3. The square which was painted red and white or vice versa looked pink in colour.

Thus, the squares on the board appeared to be of different colours. You write down the name of the colour of each square in it.

Fig. 5

A	B	C
D	E	F
G	H	I

TASK NO. 8

Objective

The objective of the task was to determine the extent of the development of the ability to state and test hypotheses.

However, two different measures could be taken for stating hypotheses and testing hypotheses, but here the combination of the both has been taken as the measure of the ability to state and test hypotheses.

Description

The task was based on a simple pendulum, the diagram of which was provided to the subjects in the printed form. The subjects were to state hypotheses regarding the factors upon which depends the time period (time taken for one oscillation) of the pendulum. Then they were to test these hypotheses describing the controlled experiments. This was an open-ended task. Scoring was done awarding one score for each correct statement of a hypothesis and one score for describing a

controlled experiment to test the same. The maximum score on this task was found to be 9 and the minimum as zero.

Administration

Then asked Mohan what are the factors upon which depends the drying up of a wet handkerchief? Mohan told her that this question could have many answers, such as :

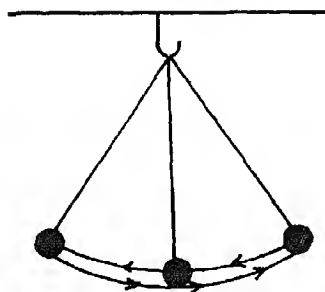
1. Length; 2. Thickness; 3. Colour;
4. Temperature; 5. Season;
6. Nature of the stuff, i.e., cotton, silk, etc.

Then she asked Mohan to prove the effect of these factors with the help of the experiments. Mohan stated as follows:

Suppose I am to prove that the drying up of a wet handkerchief depends upon the length of it. I shall take three handkerchieves of the same stuff, same thickness, same colour, etc., but with different lengths. I shall make them equally wet and put them in the sun or shade. The time taken by each handkerchief to dry up will be noted with the help of a watch. If the handkerchief having the smallest length dries up first and the one having the largest length dries up at the last, then it is proved that the drying up of a wet handkerchief depends upon the length of it, or otherwise not. Similarly, the effect of the other factors can be proved through the experiments.

Now you please solve the problem given below:

Fig. 6



A simple pendulum is shown in Fig. 6 above. The bob of the pendulum oscillates on both sides of the centre. The movement of the bob from the centre to left end, back to the centre, then to the right end and back to the centre is called the one oscillation of the pendulum. You write down the factors

upon which depends the visualization in one oscillation of the pendulum and prove the effect of each factor through experiments.

TASK NO. 9

Objective

The purpose of the task was to assess the ability of the subjects to visualize the space critically. The performance on the task has been designated as the measure of the space visualization.

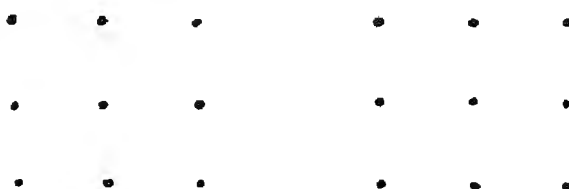
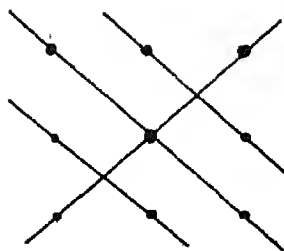
Description

This task consisted of a number of sets consisting of nine dots (eight on the four sides of a square and one at the centre). The subjects were required to link all the nine dots with four straight lines only. They were to see that no dot should remain away from the lines and the number of lines should not exceed four in any case. It was an open-ended task. One score was awarded for each correct solution. The maximum score reached on this task was 30 and the minimum was zero.

Administration

A number of sets of nine dots are given below (Fig. 7).

Fig. 7



You please try to draw four straight lines in such a way that all the nine dots are touched by one or the other line. Repeat this exercise in as many different ways as you can but the number of lines should not exceed four and no dot should remain untouched.

TASK NO. 10

Objective

The task was designed to find out if the subjects at formal-operational stage can grasp the essence of the problem. This ability is expected to be developed till adolescence. The measure based on the performance on this task represents the ability to grasp the essence of the problem.

Description

This task consisted of five statements which were designed to test whether the subjects could grasp the essence of the problem if posed in a ticklish style. The problems involved in the statements were very simple and easy but they were a bit vexatious question. Thus, if the subjects could sense or grasp the intrinsicity involved then they could reach the solution correctly. One score was awarded for each correct solution. The maximum score on this task came out to be 5 and the minimum as zero.

Administration

Read, understand and then answer the questions given below:

1. There is a 10 metre long rod of wood out of which 1 metre rod is cut after every minute. How much time will it take to be cut into pieces of 1 metre length each?

2. Some ducks are swimming in a pond in a straight line. Two ducks are on the front side, two in the middle and two on the backside. How many ducks are there in all?
3. Ham has four friends. Three of his friends are having names as Bryan, Helen and Tilly. What is the name of the fourth friend?
4. There is a blind man. He can see upto 100 metres through one eye. How far will he be able to see through both the eyes?
5. A donkey has two horns. How many horns will be having eight donkeys?

Reliability and Validity of the Test of Piaget Type Tasks (Task-wise)

The test re-test reliability coefficients for all the tasks were determined on the basis of re-administration of the test after two months to a group of 50 students. The validity coefficients were worked out for all the ten tasks against the measures of verbal intelligence, non-verbal intelligence and reasoning ability. The results are presented in Table 1.

TABLE 1

COMPUTATION OF RELIABILITY AND VALIDITY OF
A BATTERY OF PIAGET TYPE TASKS (PART-II)

Task Nos.	N = 50	N = 906		
	Test Retest Reliability Co-efficient.	Validity Coefficients Against External Criterion		
		Intelligence Verbal	Intelligence Non-verbal	Reasoning Ability
Task No. 1	.53	.30	.40	.29
Task No. 2	.21	.45	.41	.37
Task No. 3	.33	.54	.53	.42
Task No. 4	.80	.33	.46	.43
Task No. 5	.60	.39	.29	.32
Task No. 6	.40	.42	.32	.36
Task No. 7	.52	.56	.48	.43
Task No. 8	.62	.50	.37	.42
Task No. 9	.63	.30	.33	.26
Task No. 10	.22	.20	.23	.13

Reliability of the Test of Piaget Type Tasks (Combined)

As suggested by Schwarz and Krug (1972) that the tests which overlap each other provide two separate estimates of the ability common to both and thereby increase the overall accuracy of its appraisal. Thus even the tests of moderate reliability can in combination yield a highly reliable measure of the same. They have suggested that the reliability of the tests in combination can be computed from the individual reliabilities and the inter-test correlations as per following steps:

- Step I Preparation of matrix of tests' reliabilities and inter-correlations.
- Step II Sum of all the elements included in the matrix.
- Step III Replacement of reliability coefficients of individual tests with the value 1.00 in the diagonal entries and sum the matrix again.
- Step IV Division of first sum by the second sum mentioned above.

The resultant figure provides the quotient of the estimated reliability of the test in combination.

The results obtained through this method regarding the reliability of the tasks in combination are presented in Table 2.

TABLE 2

COEFFICIENT OF RELIABILITY OF THE SET OF
PIAGET TEST TASKS (COMBINED)

	1	2	3	4	5	6	7	8	9	10
Task No. 1	.53	.34	.42	.33	.19	.33	.36	.30	.31	.24
Task No. 2	.34	.21	.42	.30	.23	.33	.46	.34	.21	.24
Task No. 3	.42	.42	.35	.45	.30	.43	.51	.43	.34	.23
Task No. 4	.33	.39	.43	.50	.28	.46	.46	.43	.36	.21
Task No. 5	.19	.23	.30	.28	.50	.19	.27	.28	.18	.16
Task No. 6	.33	.33	.43	.46	.19	.43	.33	.42	.35	.21
Task No. 7	.36	.46	.51	.46	.27	.33	.52	.40	.25	.19
Task No. 8	.30	.34	.43	.43	.28	.42	.40	.62	.23	.18
Task No. 9	.31	.21	.34	.36	.18	.35	.25	.23	.63	.19
Task No. 10	.24	.24	.23	.21	.16	.21	.19	.18	.19	.22

Sum of the 100 entries written in the above matrix = 33.24

Sum of the 100 entries after replacing the diagonal entries with 1.00 = 38.28

Coefficient of reliability of the tasks in combination $\gamma = \frac{33.24}{38.28} = .87$

Thus the value of coefficient of reliability was found to be sufficiently high which proved that the test was quite dependable.

Validity of the Test of Piaget Type Tasks (Combined)

The procedure to work out the validity of the test in combination was very much similar to that described earlier for estimating the reliability of the tests in combination. Schwars and Brug (1973) has suggested the following steps for the computation of the combined validity:

- Step I Preparation of the matrix of tests' validities in the diagonal entries and the inter-test correlations.
- Step II Sum all the validity co-efficients written in the diagonal entries.
- Step III Replacement of validity co-efficients of individual tests with the value 1.00 in the diagonal entries and sum the matrix again. Computation of the square root of this sum.
- Step IV Division of first sum by the square root of the second sum.

The resultant value provides the estimate of validity of the tests in combination. The validities, thus, calculated for the combined tasks against the measures of verbal intelligence, non-verbal intelligence and reasoning ability were found to be .69, .63 and .55 respectively which are presented in Tables 3,4 and 5 respectively.

TABLE 3

VALIDITY OF "A TEST OF PILOT TYPE TEST"
(INDIVIDUAL) AGAINST VERBAL INTELLIGENCE

	1	2	3	4	5	6	7	8	9	10
Task No. 1	.30	.34	.42	.33	.19	.33	.36	.30	.31	.24
Task No. 2	.34	.43	.42	.30	.23	.33	.46	.34	.21	.24
Task No. 3	.42	.42	.54	.43	.30	.43	.51	.43	.34	.23
Task No. 4	.33	.39	.45	.53	.23	.46	.46	.43	.36	.21
Task No. 5	.19	.23	.30	.21	.39	.19	.27	.20	.18	.16
Task No. 6	.33	.33	.43	.46	.19	.42	.33	.42	.33	.21
Task No. 7	.36	.46	.51	.46	.27	.33	.36	.40	.25	.19
Task No. 8	.30	.34	.43	.43	.21	.42	.40	.50	.23	.18
Task No. 9	.31	.21	.34	.36	.18	.35	.23	.23	.30	.19
Task No. 10	.24	.24	.23	.21	.16	.21	.19	.19	.19	.20

Sum of validity coefficients written
in the diagonal entries

= 4.27

Sum of the 100 entries after replacing the
diagonal entries with 1.00

= 38.23

Square root of the second sum

= 6.187

Coefficient of validity of the tasks in
combination against the criterion of
verbal intelligence

γ = .69

TABLE 4

VALIDITY OF P. L. TEST OF MENTAL XEN. 1 TASKS
(CRITERION) AGAINST NON-VERBAL INTELLIGENCE

	1	2	3	4	5	6	7	8	9	10
Task No. 1	.41	.34	.42	.33	.19	.33	.36	.21	.31	.24
Task No. 2	.34	.41	.42	.39	.33	.33	.46	.34	.21	.24
Task No. 3	.42	.42	.51	.45	.30	.45	.51	.43	.34	.23
Task No. 4	.33	.39	.43	.46	.31	.46	.46	.43	.36	.21
Task No. 5	.19	.33	.30	.31	.29	.19	.37	.21	.18	.16
Task No. 6	.33	.33	.43	.46	.19	.32	.31	.42	.35	.21
Task No. 7	.36	.46	.51	.46	.27	.33	.41	.40	.23	.19
Task No. 8	.21	.34	.43	.43	.21	.42	.40	.37	.23	.18
Task No. 9	.31	.21	.34	.36	.18	.33	.25	.23	.33	.19
Task No. 10	.24	.24	.23	.21	.16	.21	.19	.14	.19	.23

Sum of validity coefficients written
in the diagonal entries

= 3.91

Sum of the 100 entries after replacing
the diagonal entries with 1.00

= 38.20

Square root of the second sum

= 6.187

Coefficient of validity of the tasks in
combination against the criterion
of non-verbal intelligence.

γ = .62

TABLE 5

VALIDITY OF THE TEN JOINTLY VALID TASKS
(CORRELATED) AGAINST REASONING ABILITY

	1	2	3	4	5	6	7	8	9	10
Task No. 1	.39	.34	.42	.33	.19	.33	.36	.39	.31	.24
Task No. 2	.34	.37	.43	.30	.23	.33	.46	.34	.21	.24
Task No. 3	.42	.42	.43	.47	.30	.43	.31	.43	.34	.23
Task No. 4	.33	.30	.43	.45	.21	.46	.46	.43	.36	.21
Task No. 5	.19	.23	.30	.21	.32	.19	.27	.21	.18	.16
Task No. 6	.33	.33	.43	.46	.19	.36	.33	.42	.35	.31
Task No. 7	.36	.46	.51	.46	.27	.33	.43	.40	.25	.19
Task No. 8	.39	.34	.43	.43	.21	.42	.40	.42	.23	.18
Task No. 9	.31	.21	.34	.36	.18	.38	.25	.25	.26	.19
Task No. 10	.24	.24	.23	.21	.16	.21	.19	.18	.19	.13

Sum of the validity coefficients
written in the diagonal entries = 3.45

Sum of the 100 entries after replacing
the diagonal entries with 1.00 = 39.28

Square root of the second sum = 6.167

Coefficient of validity of the tasks
in combination against the criterion
of reasoning ability $Y = .56$

CHAPTER 3

Description of the Other Tests Used

In addition to the Test of Piaget Type Tasks described earlier, some other standard psychological tests were also included in the study to explore the allied dimensions of formal-operational thought and to make the process of interpretations of factors more sound. The description of these tests are given below:

Cattell's Culture Fair Intelligence Test (Scale 3)

The purpose of the test, according to the authors, is to provide a single best measure of intelligence as the modern basis for the general intelligence having the highest validity on Thurstone's second order general ability factor or Spearman's 'g' (general mental capacity). The test, therefore, deals with the core of general relation education capacity which many researchers have shown to be largely inborn, a relatively constant characteristic for the individual and operative in quite different fields of content, e.g., verbal, numerical, spatial and social skills. The authors claim that the test is highly suitable for the varied research situation, especially for those in which general ability is the variable to be operated. It measures intelligence in separation from chance educational influences and local social climate.

The scale consists of 46 problems in all, covered under 4 sub-tests, i.e., series (12), classifications (14), matrices (12) and conditions (8). In all the sub-tests, the items are arranged

in order of increasing difficulty. It is available in two equivalent forms, A and B. This assumes the standard 'speeded test' administration in which the time limit is accurately adhered to for each sub-test as 3,4,4 and 2.5 minutes respectively. Scale 2 is designed for eight through fourteen year olds and for unselected (non-college) adults. It may be used both as an individual and as a group test.

Authors have reported that the reliability of the scale, both in terms of the Dependability Coefficient and the Consistency Coefficient, was evaluated. The Dependability Coefficient (immediate test re-test agreement) on the full test was found to be ranging from .82 to .85. The Consistency Coefficient (split-half, corrected to full length using both A and B forms) for four different groups was ranging between .70 and .92. The validity of the test can be envisaged from the fact that it has given 'r' values of .36 to .71 with the Revised Stanford Binet, of .73 with the Otis 'Quick Scoring' test and of .59 with A.C.T.. The measure of intelligence (non-verbal) based on the scores obtained on this test has been denoted by the code IW.

Jalota's General Mental Ability Test

It is a group test of general mental ability which can be administered to Hindi knowing school children. The test consists of 100 items distributed over the elements of Vocabulary - Similar (10), Vocabulary-Opposites (10), Number-Series (20), Classification (20), Best Answers (10), Inferences (10) and Analogies (20). The items are presented on a homogeneous format, i.e., the items belonging to different elements are inter-

arranged. The time allowed to answer 100 items is 30 minutes. The reliability of the test has been reported to be .938. The validity coefficients of the test with respect to school examination marks ranged from .50 to .70. The measure of intelligence based on the performance on this test is given the code IV.

Jubay's Reasoning Ability Test

The author has claimed that the test is a good measure of reasoning ability associated with problem solving. The test contains 60 items. First 40 items are number series in which last two spaces are to be filled in by the subjects. Thus, these items have two answers for each item and hence carry 80 scores. The next 20 items represent problems, and each carries one mark for correct answer. The time limit for the test is 60 minutes. It is meant for students knowing Hindi and of 12-17 years of age. The reliability coefficients of the test have been reported as .886 and .913 for National Equivalence Method and Split-half Method respectively. The validity coefficients of the test with respect to the external criteria such as Group Test of Intelligence (R.T. Tandon), Progressive Matrices and Problem Solving Ability Test have been reported to be .875, .836 and .882 respectively.

Asthana's Adjustment Inventory

The inventory attempts to segregate the poorly adjusted from those who are better adjusted. It works as a quick screening device for use with the Hindi knowing school and college students. The inventory consists of 40 items excluding

items 29 and 34. It has no fixed time limit. It takes about 30 minutes to answer all the items. The inventory is self-administering. The examinees are to interpret the questions for themselves. However, the meanings of the difficult words were explained by the investigator to the younger subjects. The scoring was done awarding one score for 'No' and zero for 'Yes' response except for items 19 and 40 where it was in the reverse order as described by the author. The reliability coefficient by split-half method has been reported to be .97. The validity index of the test items determined through bi-serial correlation technique using total test score as criterion measure has been reported to be satisfactory.

Cattell's High School Personality Questionnaire (HPQ)

The HPQ is a standard instrument that gives an objective analysis of fourteen distinct dimensions of personality.* The author has a claim that these fourteen dimensions have been found to be covering almost the total personality. Form A of the Hindi version of the HPQ (Kapoor and Mehrotra, 1967) was used in the study under report. It consists of 114 items, all of multiple choice. The reading level of the test is adapted to ages 11 or 12 through 18 years. No time limit has been suggested but all the students were able to complete it in about 30 minutes. The immediate test re-test reliability coefficients (dependability

* For the description and the codes of the fourteen dimensions of personality as well as other variables, see appendix (iv).

coefficients) for the fourteen dimensions have been reported to be ranging from .74 to .91 and test retest reliability coefficients after six months period (stability coefficients) range from .53 to .69. The coefficients of validity (construct) range from .57 to .77 for the fourteen dimensions.

Space Relations Test (RT)

The Differential Aptitude Tests' battery was developed by Bennett, et al (1939). The battery was designed for use with high school students. The Differential Aptitude Test consists of eight tests designed to measure eight different abilities : verbal reasoning, numerical ability, space relations, abstract reasoning, clerical speed and accuracy, mechanical reasoning, language usage and spelling and sentences. In the study at hand only the space relations test was used. It measures abilities to visualize a constructed object from a pattern and to manipulate a form in order to judge its appearance after rotation in various ways. The space relations test consists of 60 items only. The testing time for the test is 35 minutes. Split-half reliability of the test has been reported to be .93 by the authors. Validity coefficient of the test against the science grades was lying between .40 and .50.

Academic Achievement in School Subjects

The measures of academic achievement in five school subjects, i.e., Mathematics, Science, English, Punjabi and Hindi were based on the scores of the students obtained at examinations conducted by the schools. Since the standard of examination varies from school

to school, therefore, the scores of all the five subjects were converted into standard scores for each class. The scores of all the five subjects were kept separate as it is understood that each subject requires a specific sort of psychic abilities. The separate scores of the different subjects may also be helpful in the identification and explanation of the factors extracted through factor analysis. The examination scores of five school subjects were treated just as the performance on a battery of five tests and hence the reliability of the overall measure of academic achievement was worked out, from the average coefficient of inter-subject correlations which came out to be .44, by applying the formula :

Reliability of the overall measure
of academic achievement

$$r = \frac{nr'}{1 + (n-1)r'}$$

where n = number of tests taken

r' = average coefficient of inter-
subject correlations.

The coefficient of reliability of the overall measure of academic achievement was found to be .80. The validity coefficients for the overall measure of academic achievement were found to be .36, .32 and .30 respectively against the external criteria of verbal intelligence, non-verbal intelligence and reasoning ability respectively when computed by the method described earlier for the validity of the Test of Piaget Type Tasks.

Chapter V

Results of Descriptive Statistics
and
Bivariate Analysis

CHAPTER V

Results of Descriptive Statistics and
Bivariate Analysis

Section A

Results of Descriptive Statistics

It was thought desirable to present the results of the descriptive statistics in terms of mean, median, mode, standard deviation, standard error, kurtosis and skewness regarding the various measures included in this study before presenting the actual results related to the hypotheses as the former may be considered good indicators of the attributes essential for the fulfilment of the assumptions underlying the theoretical framework of the statistical techniques employed for the analysis of data. These statistics were computed, using the programme *STATISTICAL with ALL APPLICATIONS* from the Statistical Package for the Social Sciences (Nie, et al., 1970), through the 'ASAP - 1022 Computer' at Computronics India, New Delhi. To furnish a clear picture of the distributions of the measures of personality, academic achievement, adjustment, intelligence, space relations, reasoning ability and adolescent thought, the results regarding the descriptive statistics have been presented in the tabular form as follows:

TABLE 6

THE VALUES OF MEAN, MEDIAN, MODE, STANDARD DEVIATION,
STANDARD ERROR, KRIPPON'S AND ALLIANCE FOR FORMS
ON DIMENSIONS OF PERSONALITY (199)

Dimension of Perso- nality	Mean	Median	Mode	SD	SE	Ku	Al
A	9.495	9.604	10.000	2.596	.083	.036	-.083
B	3.134	3.076	3.000	1.398	.045	.144	.273
C	8.754	8.659	8.000	3.697	.086	-.246	-.006
D	7.665	7.737	8.000	3.751	.088	-.217	-.015
E	7.333	7.558	8.000	2.552	.081	-.207	-.073
F	8.902	9.043	10.000	2.571	.082	-.011	-.171
G	9.718	9.723	8.000	2.698	.086	-.237	-.016
H	8.512	8.404	8.000	2.919	.093	-.153	-.009
I	7.805	7.873	8.000	2.743	.087	-.108	-.041
J	7.743	7.783	8.000	2.521	.080	-.210	.008
K	6.818	6.800	8.000	2.315	.093	-.475	.006
L ₁	7.548	7.756	8.000	2.750	.088	.020	-.155
L ₂	8.933	8.907	10.000	2.797	.089	-.002	-.068
L ₃	7.266	7.500	6.000	2.803	.089	-.270	-.095

The results presented in Table 6 make it quite clear that the values of mean and median for all the dimensions of personality were in close agreement with each other. The values of SDs were found to be varying between 2.521 and 2.919 for all the measures except the one (B) whose SD was 1.398. The values of standard errors of mean were found to be ranging from .045 to .093 which

demonstrated the stability of all the measures of personality. The values of kurtosis were found to be varying between $-.002$ and $+.475$ for the eleven dimensions ($1_1, 1_2, 1_3, 1_4, 1_5, 1_6, 1_7, 1_8, 1_9, 1_{10}$ and 1_{11}) and for the remaining three dimensions ($4_1, 4_2$ and 4_3) between $.020$ and $.144$. The indices of skewness were found to be ranging between $-.171$ and $.273$. Since the indices of skewness and kurtosis were having small magnitudes in the case of the majority of the measures and the values of mean and median showed a good harmony with one another for all the measures, the measures of personality may be considered as normally distributed.

TABLE 7

THE VALUES OF MEAN, MEDIAN, MODE, STANDARD DEVIATION, VARIATION COEFF, KURTOSIS AND SKEWNESS FOR FIVE MEASURES OF ACADEMIC ACHIEVEMENT

Measure of Achievement	Mean	Median	Mode	SD	SE	Ku	Sk
AMT	50.229	50.234	47.000	9.598	.306	-.506	.016
AAS	50.305	50.419	48.000	9.693	.309	-.474	-.054
AAE	50.523	50.405	49.000	9.433	.301	-.468	.013
AAI	50.373	50.183	48.000	9.485	.302	-.432	-.000
AAT	50.443	50.441	52.000	9.564	.305	-.442	.023

It is evident from the results presented in Table 7 that for all the five measures of academic achievement the values of mean and median were identical (lying between 50.183 and 50.523).

The values of SD_s were found to be ranging between 9.433 and 9.693. The standard errors of mean ranged from .301 to .309 for the five measures. The values of kurtosis were found to be lying between -.432 and -.506, and thus the measures were leptokurtic. The indices of skewness were ranging between -.054 and .022 which demonstrated that the measures of academic achievement were evenly distributed.

TABLE 8

THE VALUES OF MEAN, MEDIAN, MODE, STANDARD DEVIATION, STANDARD ERROR, KURTOSIS AND SKEWNESS FOR THE MEASURES OF ADJUSTMENT, VERBAL INTELLIGENCE, NON-VERBAL INTELLIGENCE, SPACE RELATION AND REASONING ABILITY.

the Measure	Mean	Median	Mode	SD	SE	Kurtosis	Skewness
ADJ	20.733	20.894	18.000	5.443	.173	.250	-.133
IV	19.476	17.833	17.000	9.068	.314	2.367	1.084
NV	16.144	15.764	13.000	5.515	.176	.020	.250
RI	24.488	24.069	25.000	8.261	.263	.039	.254
RA	12.664	11.567	10.000	5.373	.171	10.325	2.163

The results presented in Table 8 have established that the measure of adjustment (ADJ) was normally distributed, as the values of mean (20.733) and median (20.894) were found to be identical, and the indices of skewness (-.133) and kurtosis (.250) were not deviated from the normal values considerably. The values of IV and NV were found to be 5.443 and .173 respectively.

The measure of verbal-intelligence (IV) was found to be positively skewed as the median (17.833) was lying to the left of the mean (19.476) and the value of skewness was found to be 1.084 (vide Table 8). The measure had β_1 equal to 9.868 and β_2 of mean equal to .314. The measure was platykurtic as the value of kurtosis was found to be 2.367.

It is evident from the results presented in Table 8 that the distribution of the measure of non-verbal intelligence (ENV) was normal, as the values of mean (16.144) and median (15.764) were in close vicinity of each other, and the values of kurtosis (.020) and skewness (.250) were also close to the normal values. The values of β_1 and β_2 were found to be 5.513 and .176 respectively.

Looking through the results presented in Table 8, it was found that the values of mean (24.433) and median (24.069) of the measure of space relations (SI) were in close agreement with each other, and the values of kurtosis (.069) and skewness (.264) were also within the acceptable limits. Hence the distribution of the measure may be considered normal. The β_1 of the measure was found to be 5.261 and the β_2 of mean as .263.

The results presented in Table 8 show that the measure of reasoning ability (RA) was positively skewed, as the value of median (11.867) was found to be lying to the left of the value of mean (12.664), and the index of skewness was found to be 2.163. The value of kurtosis was found to be 10.326 which showed that the measure was platykurtic. The values of β_1 and β_2 were found to be 5.373 and .171 respectively.

TABLE 9

VALUES OF MEAN, MEDIAN, MODE, STANDARD DEVIATION,
STANDARD ERROR, COEFFICIENT AND SKEWNESS FOR
THE TEN MEASURES OF ADOLESCENT THOUGHT

Measure of Adolescent Thought	Mean	Median	Mode	SD	SE	Sk	Sk
CL	33.590	33.346	34.000	18.093	.576	-1.199	-.335
GT	2.441	2.407	.000	1.833	.088	-.863	.268
GAA	11.292	11.304	.000	6.995	.223	-1.199	-.039
PAC	2.423	1.715	.000	2.625	.084	.819	1.738
RA	1.170	0.000	.000	2.152	.069	1.779	1.791
FPQ	5.453	5.211	.000	4.236	.135	-.180	.533
ICI	3.739	2.843	2.000	2.987	.095	-.992	.582
STI	1.829	1.547	.000	1.862	.059	.738	.962
W	8.439	7.954	9.000	4.751	.151	.790	.801
GA	.977	.764	.000	1.093	.035	.805	1.019

The results presented in Table 9 demonstrate that the values of mean and median for the measures of adolescent thought were in close agreement with each other except in the case of the three measures, namely, permutations and combinations (PAC), ratio and proportion (RA), and grasping the essence of the problem (GA) where the variations were quite marked. The values of skewness were found to be ranging from -.335 to 1.791. The measure of classification (CL) was negatively skewed; the measures of permutations and combinations (PAC), ratio and proportion (RA), formulation of probing questions (FPQ), interpretation and coordination of information (ICI), stating and testing of

hypothesis (Q_1), space visualisation (Q_5) and grasping the essence of the problem (Q_2) were positively skewed; and the measures of grouping of thought (Q_3) and generalization to arithmetical and algebraic symbols (Q_4) were normal. The indices of kurtosis were found to be varying from -1.999 to 1.779. The measures of classification (Q_6), grouping of thought (Q_3), generalisation to arithmetical and algebraic symbols (Q_4) and interpretation and coordination of information (Q_7) were leptokurtic and the measures of pre-permutations and combinations (Q_8), ratio and proportion (Q_9), stating and testing hypotheses (Q_{10}), space visualization (Q_5) and grasping the essence of the problem (Q_2) were platykurtic and the measure of formulation of probing questions (Q_{11}) was mesokurtic.

A close view of the nature of the distributions of the various measures described in the preceding section reveals that in most of the cases the distributions were normal with little variations. Thus for the further statistical analysis all the measures have been assumed to be normally distributed with respect to the population under study.

Section B

Results of Bivariate Analysis

The data were analysed through bivariate analysis in order to test hypotheses 1 to 7. The results are presented below as per sequence of the hypotheses stated earlier.

The first hypothesis of the study was : Does the performance on Piaget Type Tasks increase with age during the formal-operational period? The data regarding the performances on various Piaget Type Tasks at different age levels (11⁺, 12⁺, 13⁺, 14⁺ and 15⁺ years) were put to one way analysis of variance to determine the significance of the variations in the performances on Piaget Type Tasks at different age levels. The F-ratios computed for all the ten Piaget Type Tasks have been presented in Table 10.

It is evident from the results presented in Table 10 that all the F ratios were significant at $<.01$ level which proved that significant differences exist between the performance of the students at 11⁺, 12⁺, 13⁺, 14⁺ and 15⁺ age levels for the tasks of classification (CL), grouping of thought (GT), generalisation to arithmetical and algebraic symbols (GAS), permutations and combinations (PAC), ratio and proportion (RAP),

TABLE 10

RELATIONSHIP FOR THE ANALYSIS OF VARIANCE ATTACHED TO FIVE AGE
LEVELS AND THE PROPORTIONS ON EACH OF THE TEN DIETARY TASKS

Task	Variable	Sum of Squares	Degrees of Freedom	Mean Square	F	Total S.S. Proportion
CL	Between Groups	32705.750	4	8176.437	34.569	<.01
	Within Groups	232341.251	981	237.809		
GOT	Between Groups	841.534	4	210.383	83.160	<.01
	Within Groups	2512.013	981	2.561		
GAA	Between Groups	13872.227	4	3468.057	93.199	<.01
	Within Groups	34925.541	981	35.502		
PAJ	Between Groups	1891.336	4	472.834	73.145	<.01
	Within Groups	5193.515	981	5.294		
RAP	Between Groups	527.383	4	131.833	32.051	<.01
	Within Groups	4034.022	981	4.112		
TPJ	Between Groups	4395.575	4	1098.669	80.955	<.01
	Within Groups	13297.587	981	13.545		
ICI	Between Groups	2332.273	4	583.068	88.602	<.01
	Within Groups	5455.745	981	5.561		
SPJ	Between Groups	279.550	4	69.888	85.110	<.01
	Within Groups	2534.434	981	2.584		
SV	Between Groups	2529.418	4	632.354	27.745	<.01
	Within Groups	22446.705	981	22.881		
QSP	Between Groups	45.227	4	11.307	9.530	<.01
	Within Groups	1151.772	981	1.174		

formulation of probing questions (FPQ), interpretation and coordination of information (ICI), stating and testing hypotheses (STH), space visualisation (SV) and grasping the essence of the problem (GEP).

The dimensions of adolescent thought included in this study show a trend of growth during the formal operational period (vide Table 11). Though in the beginning it was mentioned that some of the tasks related to ^{the} concrete-operational stage such as tasks of classification and grouping of thought were included just to maintain continuum of the measures of adolescent thought, but the findings of this study revealed that even these dimensions had not reached the state of perfection till late adolescence. Thus the earlier findings of the research workers (Dale, 1970; Dulit, 1972; Higgings and Gaite, 1971; Keatings, 1975; Lawson and Renner, 1974; Lawson and Blöte, 1975; Renner and Stafford, 1972) that not only high school students but college students and in some cases teachers also were found operating at concrete-operational stage, seems quite convincing in the light of the results of the present study. The positive skewness in the case of the measures of ratio and proportion (RAP), grasping the essence of the problem (GEP), stating and testing hypotheses (STH) and space visualisation (SV) does indicate that the majority of the scores on these measures of adolescent thought were lying on the lower side of the mean of the scale (vide Table 9), which implicitly demonstrate that these dimensions have not been developed substantially among the subjects tested.

TABLE 11

MEAN SCORE AND STANDARD DEVIATION OF JACO
TASK AT DIFFERENT AGE LEVELS

Piglet Type Task	Age in Years					Overall Total
	11 ⁺	12 ⁺	13 ⁺	14 ⁺	15 ⁺	
	N = 160	205	225	212	175	986
CL	22.125 (16.403)	30.815 (17.076)	34.327 (17.711)	39.363 (16.403)	40.749 (16.892)	33.740 (10.084)
OOT	1.006 (1.323)	1.707 (1.547)	2 .531 (1.804)	3.200 (1.671)	3.637 (1.534)	2.445 (1.843)
OAA	5.744 (5.470)	8.639 (5.936)	10.735 (6.555)	15.259 (5.610)	15.606 (6.069)	11.202 (6.995)
PAC	.631 (1.561)	1.342 (1.871)	2.297 (2.234)	3.660 (2.566)	4.086 (2.994)	2.425 (2.625)
RAV	.232 (.853)	.595 (1.720)	.898 (1.832)	2.094 (2.614)	1.977 (2.560)	1.170 (2.152)
W77	2.202 (3.006)	3.620 (3.120)	6.120 (3.961)	6.670 (3.734)	8.399 (4.240)	5.453 (4.236)
ICI	1.577 (1.796)	2.454 (2.299)	3.504 (2.689)	5.443 (2.953)	5.860 (2.260)	3.739 (2.987)
STH	.494 (.882)	1.039 (1.271)	1.766 (1.590)	2.830 (1.880)	3.205 (2.091)	1.829 (1.862)
SV	5.691 (3.869)	7.971 (4.734)	8.287 (4.632)	10.453 (4.543)	9.874 (4.789)	8.520 (5.037)
GSF	.679 (.968)	1.005 (1.080)	.960 (1.144)	.962 (.927)	1.306 (1.236)	.999 (1.108)

Standard Deviations are given in brackets.

It is clear from the results presented in Table 11 that the performances of the students at different age levels (11⁺, 12⁺, 13⁺, 14⁺ and 15⁺ years) show ^{an} increasing trend for almost all the tasks, which is more evident from the Fig. 8.

Thus, it has been confirmed that the students of the age group 11 to 15 years have not reached the formal-operational level to its full as they have been found to be still growing. It has been concluded that performance on Piaget Type Tasks increases with age during the formal-operational period. The first hypothesis, thus, stands empirically verified and declared affirmatively.

Regarding the second hypothesis: whether boys and girls perform equally well on Piaget Type Tasks ?, the performances of boys and girls on Piaget Type Tasks at different age levels as well as for the combined groups were analysed using t-test technique to determine the significance of the difference between them. The results, thus computed, regarding the performances on ten Piaget Type Tasks have been presented in Tables 12 to 21.

It is evident from the results presented in Table 12 that sex differences were quite obvious at age levels 11⁺, 12⁺ and 13⁺ where boys have shown better performance than girls on the task of classification (CL). At 14⁺ and 15⁺ age levels no significant difference was found between the performance of boys and girls. Thus, it is very interesting to note that the

TABLE 12

RESULTS REGARDING THE COMPARATIVE PERFORMANCE OF BOYS
AND GIRLS ON THE TASK OF CLASSIFICATION AT FIVE
AGE LEVELS AS WELL AS FOR THE COMBINED GROUP

Age level in Yrs.	Sex	N	Mean Perfor- mance	SD	Difference of Means (D)	Z_p	t	Level of Signi- ficance
11 ⁺	B	88	34.909	16.011	5.846	2.520	2.320	*
	G	80	19.063	16.581				
12 ⁺	B	105	34.914	17.272	8.404	2.313	3.633	**
	G	100	26.510	15.840				
13 ⁺	B	116	37.716	17.072	6.961	2.318	3.003	**
	G	110	30.755	17.740				
14 ⁺	B	103	39.925	16.533	1.147	2.259	.503	n.s
	G	104	38.779	16.445				
15 ⁺	B	88	41.552	17.092	2.220	2.531	.870	n.s
	G	87	39.632	16.650				
Com- bined	B	505	36.095	17.625	4.827	1.143	4.223	**
	G	481	31.252	18.184				

* Significant at .05 level

** Significant at .01 level

n.s Not significant

TABLE 13

RESULTS REGARDING THE COMPARATIVE PERFORMANCE OF
BOYS AND GIRLS ON THE PATH OF THOMPSON OF 2 D M E
AT THE FIVE AGE LEVELS AS WELL AS FOR THE COMBINED
GROUP

Age Level in Yrs.	Sex	N	Mean Perfor- mance	SD	Difference of Means (t)	SD	t	Level of Signi- ficance
11 ⁺	B	88	.932	1.192	-.156	.207	.755	n.s
	G	80	1.000	1.460				
12 ⁺	B	105	1.762	1.451	.112	.216	.514	n.s
	G	100	1.650	1.643				
13 ⁺	B	116	2.612	1.883	.156	.240	.692	n.s
	G	110	2.445	1.722				
14 ⁺	B	108	3.259	1.672	.125	.230	.544	n.s
	G	104	3.144	1.675				
15 ⁺	B	88	3.830	1.533	.347	.251	1.508	n.s
	G	87	3.493	1.524				
Com- bined	B	505	2.485	1.853	.102	.117	.859	n.s
	G	481	2.393	1.527				

n.s Not significant.

TABLE 14

RESULTS REGARDING THE COMPARATIVE RECOGNITION OF DEPTHS AND
GENTS ON "H" TASK OF GENERALIZATION TO STATE PLACING THE
ARITHMETIC SYMBOL AT THE FIVE AGE LEVEL AS SET AS FOR
THE COMBINED GROUPS

Age Level in Yrs.	Sex	N	Mean Perfor- mance	(S)	Difference of Means (D)	S.D	t	Level of Signi- ficance
11 ⁺	B	88	5.693	5.476	-.107	.640	.126	n.s
	G	90	5.800	5.515				
12 ⁺	B	106	9.543	5.786	1.813	.823	2.203	*
	G	100	7.730	6.008				
13 ⁺	B	116	11.379	6.493	1.324	.871	1.521	n.s
	G	110	10.055	6.587				
14 ⁺	B	103	15.991	5.559	1.473	.755	1.933	n.s
	G	104	14.519	5.590				
15 ⁺	B	89	16.952	5.164	2.714	.898	3.023	**
	G	87	14.241	6.519				
Com- bined	B	505	11.954	6.985	1.379	.444	3.105	**
	G	491	10.005	6.944				

* Significant at .05 level.

** Significant at .01 level.

n.s Not significant.

TABLE 15

LETTERS PROVIDING THE DIFFERENTIAL PERFORMANCE OF BOYS AND GIRLS ON THE TASK OF PERIPHERY AND JUDGMENTS AT THE FIVE AGE LEVELS AS WELL AS FOR THE COMBINED GROUP

Age level in Yrs.	Sex	N	Mean Performance	SD	Difference of Means (D)	SD	t	Level of significance
11 ⁺	B	80	.796	1.013	.346	.334	1.477	n.s
	G	80	.450	1.023				
12 ⁺	B	103	1.548	2.035	.623	.256	2.453	*
	G	100	1.020	1.553				
13 ⁺	B	116	2.612	2.490	.640	.249	2.604	**
	G	110	1.954	1.896				
14 ⁺	B	108	4.157	2.880	1.013	.346	2.929	**
	G	104	3.144	2.459				
15 ⁺	B	88	5.080	3.029	1.999	.427	4.676	**
	G	87	3.081	2.611				
Combined	B	505	2.855	2.056	.362	.164	5.351	**
	G	491	1.973	2.251				

* Significant at .05 level.

** Significant at .01 level.

n.s Not significant.

TABLE 16

RESULTS OF ANOVA IN COMPARING TWO TYPES OF FEED
AND OF SEX, AGE, RACE OF HENS AND PROPORTION OF FEED
FIVE AGE LEVELS AS WELL AS TOTAL COMBINED GROUPS

Age level in yrs.	Sex	N	Mean Performance	SD	Difference of Means (t)	Ad	P	Level of Signi- ficance
11 ⁺	M	88	.351	.597	.061	.006	.712	n.s
	F	80	.300	.513				
12 ⁺	M	109	.319	2.070	.459	.235	1.045	n.s
	F	100	.360	1.219				
13 ⁺	M	116	1.508	2.324	1.234	.232	5.402	**
	F	110	.255	.693				
14 ⁺	M	103	2.898	2.862	1.633	.340	4.822	**
	F	104	1.350	2.019				
15 ⁺	M	88	2.796	2.801	1.547	.357	4.406	**
	F	87	1.149	1.986				
Com- bined	M	503	1.669	2.513	1.022	.132	7.676	**
	F	481	.647	1.525				

** Significant at .01 level.

n.s Not significant.

TABLE 17

RESULTS REGARDING THE OPERATIVE PERFORMANCE OF BOYS
AND GIRLS ON THE TASK OF REGULATION OF PULSING P.L. 1000
AT THE FIVE AGE LEVELS AS WELL AS FOR THE COMBINED GROUPS

Age Level in Yrs.	Sex	N	Mean Perfor- mance	SD	Difference of Means (D)	SD _D	t	Level of Signi- ficance
11 ⁺	B	88	2.693	3.302	1.030	.453	2.271	*
	G	80	1.663	2.555				
12 ⁺	B	105	4.267	3.359	1.527	.436	3.039	**
	G	100	2.940	2.870				
13 ⁺	B	116	6.586	3.915	.959	.524	1.830	n.s.
	G	110	5.627	3.967				
14 ⁺	B	108	7.855	3.237	1.825	.504	3.617	**
	G	104	5.740	3.393				
15 ⁺	B	88	9.205	4.142	1.642	.632	2.597	**
	G	87	7.563	4.220				
Com- bined	B	505	6.091	4.347	1.507	.256	4.900	**
	G	481	4.784	4.013				

* Significant at .05 level.
 ** Significant at .01 level.
 n.s. Not significant.

TABLE 18

RESULTS REGARDING THE SEPARATE PERFORMANCE OF
BOYS AND GIRLS ON "A" TASK OF INTERPOLATION AND
COMBINATION OF INFORMATION IN THE FIVE AGE GROUPS
AS WELL AS FOR THE COMBINED GROUPS

Age level in Yrs.	Sex	N	Mean Perfor- mance	SD	Difference of Means (D)	SD _D	t	Level Signi- ficance
11 ⁺	B	83	1.591	1.733	.023	.279	.100	n.s
	G	80	1.563	1.873				
12 ⁺	B	105	2.514	2.173	.124	.323	.384	n.s
	G	100	2.390	2.432				
13 ⁺	B	116	3.509	2.691	.009	.333	.025	n.s
	G	110	3.500	2.993				
14 ⁺	B	103	5.815	3.836	.757	.404	1.873	n.s
	G	104	5.050	3.036				
15 ⁺	B	88	5.909	3.659	.702	.423	1.564	n.s
	G	87	5.207	2.922				
Com- bined	B	505	3.879	2.959	.235	.190	1.507	n.s
	G	481	3.593	2.981				

n.s Not significant.

TABLE 10

EXPERIMENT RECORDING THE EXPERIMENTAL ADAPTATION OF BOYS
AND GIRLS ON THE TASK OF READING AND LISTENING
TYPOTEXT AT THE FIVE AGE LEVELS AS WELL AS FOR THE
COMBINED GROUP

Age level in Yrs.	Sex	N	Mean Perfor- mance	SD	Difference of Means (t)	SD	t	Level of Signifi- cance
11 ⁺	B	88	.443	.706	-.107	.138	.777	n.s
	G	80	.550	.970				
12 ⁺	B	108	.971	1.172	-.130	.179	.780	n.s
	G	100	1.110	1.370				
13 ⁺	B	116	1.905	1.684	.237	.210	1.368	n.s
	G	110	1.618	1.478				
14 ⁺	B	108	2.851	1.811	.772	.256	3.014	**
	G	104	2.080	1.914				
15 ⁺	B	88	3.854	1.901	1.324	.301	4.401	**
	G	87	2.540	2.073				
Com- bined	B	505	2.003	1.941	.355	.117	3.009	**
	G	481	1.647	1.750				

** Significant at .01 level.

n.s Not significant.

TABLE 20

RESULTS REGARDING THE COMPARATIVE PERFORMANCE OF BOYS
AND GIRLS ON THE TASK OF SPIRAL VITUALIZATION AT THE
SEVERAL AGE LEVELS AS WELL AS FOR THE COMBINED GROUPS

Age Level in Yrs.	Sex	N	Mean Perfor- mance	SD	Difference of Means (D)	SD _D	t	Level of Signifi- cance
11 ⁺	B	80	6.432	4.301	1.537	.580	2.643	**
	G	80	4.875	3.305				
12 ⁺	B	105	9.010	6.666	2.230	.770	3.054	**
	G	100	6.880	4.372				
13 ⁺	B	116	8.793	5.142	1.102	.613	1.790	n.s.
	G	110	7.691	4.088				
14 ⁺	B	103	11.630	4.643	2.400	.603	3.939	**
	G	104	9.231	4.120				
15 ⁺⁺	B	88	11.000	4.002	2.425	.702	3.452	**
	G	87	8.555	4.482				
Com- bined	B	505	9.434	5.430	1.873	.314	5.936	**
	G	481	7.561	4.327				

** Significant at .01 level.

n.s. Not significant.

TABLE 21

RESULTS REGARDING THE COMPARATIVE EFFECTIVENESS OF
DOIT AND STONE ON THE TASK OF GRATING T L
SECTION OF THE BRUSHES AT THE FIVE AC. LEVELS AS
WELL AS FOR THE COMBINED GROUP

Age Level in Yrs.	Sex	N	Mean Perfor- mance	SD	Difference of Means (D)	t_D	t	Level of Signifi- cance
11 ⁺	B	88	.750	1.042	.150	.148	1.015	n.s
	G	80	.600	.880				
13 ⁺	B	105	.981	1.019	-.049	.147	.335	n.s
	G	100	1.030	1.007				
13 ⁺	B	116	.957	1.130	-.007	.153	.046	n.s
	G	110	.964	1.165				
14 ⁺	B	108	1.093	1.054	.266	.135	1.965	*
	G	104	.827	.900				
15 ⁺	B	88	1.380	1.420	.373	.185	2.019	*
	G	87	1.207	.978				
Com- bined	B	505	1.063	1.163	.138	.072	1.881	n.s
	G	481	.931	1.032				

* Significant at .05 level
n.s Not significant.

effect of sex on classificatory ability disappears as the subjects attain the age of 14 years or above. However, for the combined groups of boys and girls, comprising all age levels, the performance of boys was found significantly better than the performance of girls.

The performance of boys and girls on the task of grouping of thought (G T) did not show significant difference at any age level as well as for the combined groups (vide Table 13).

The results presented in Table 14 show that the performance of boys on the task of generalization to arithmetical and algebraic symbols (GAA) was significantly better than the performance of girls at 12⁺ and 15⁺ age levels while there was no significant difference at 11⁺, 13⁺ and 14⁺ age levels. The performance of boys was significantly better than that of girls for the combined groups.

It is clear from the results presented in Table 15 that the performance of boys was significantly better than the performance of girls on the task of permutations and combinations and (P+C) at all age levels as well as for the combined groups, except at age level 11⁺ where no significant difference was found.

Regarding the performance of boys and girls on the task of ratio and proportion (RAP), no significant difference was found at the age levels 11⁺ and 12⁺ but at higher age levels,

i.e., 13⁺, 14⁺ and 15⁺ boys displayed significantly better performance than girls (vide Table 16). The performance of boys was found significantly better than girls for the combined groups as well.

In the light of the results presented in Table 17 it can be said that boys were found performing significantly better than girls, on the task of formulation of probing questions (VI), consistently at all age levels except the age level 13⁺ where no significant difference was found. They also performed better in the case of the combined groups.

Regarding the performance of boys and girls on the task of interpretation and coordination of information (VII), no significant difference was found either at any age level or for the combined groups (vide Table 18).

It is clear from the results presented in Table 19 that no significant difference between the performance of boys and girls on the task of stating and testing hypotheses (VIII) was found at the age levels 11⁺, 12⁺ and 13⁺ while the performance of boys was found to be significantly better than that of girls at the age levels 14⁺ and 15⁺ as well as for the combined groups.

The performance of boys on the task of space visualisation (IX) was found to be significantly better than the performance of girls at all age levels and for the combined groups except at the age level 13⁺ where no significant difference was found (vide Table 20).

The results presented in Table 21 demonstrate that no significant difference was found between the performance of boys and girls on the task of grasping the essence of the problem (21') at age levels 11⁺, 12⁺ and 13⁺ but the performance of boys was found to be significantly better than that of girls at age levels 14⁺ and 15⁺. However, no significant difference was found between the performance of boys and girls for the combined groups.

TABLE 22

SIGNIFICANT DIFFERENCE BETWEEN THE PERFORMANCE OF BOYS AND GIRLS ON THE PIAGET TYPE TASKS AT AGE LEVELS 11⁺ TO 15⁺ AND FOR THE COMBINED GROUPS

Age Level in Yrs.	Piaget Type Tasks									
	CL	GCT	CAR	PAI	RAI	WY	III	III	IV	CV
11 ⁺	*	-	-	-	-	*	-	-	**	-
12 ⁺	**	-	*	*	-	**	-	-	**	-
13 ⁺	**	-	-	**	**	-	-	-	-	-
14 ⁺	-	-	-	**	**	**	-	**	**	*
15 ⁺	-	-	**	**	**	**	-	**	**	*
Combined	**	-	**	**	**	**	-	**	**	-

* Significant at .05 level

** Significant at .01 level

When the results of all the ten Piaget Type Tasks were viewed as a whole with respect to the comparative performance of boys and girls, the values of thirtyone t-ratios, out of the sixty t-ratios computed for the differences of performance

of boys and girls at five age levels, i.e., 11⁺, 12⁺, 13⁺, 14⁺ and 15⁺ years as well as for the combined group, were found to be significant (vide Table 22). In all the cases of the significant t-ratios, boys were having superior performance than girls. Thus, regarding the second hypothesis, it has been concluded that boys perform either equal to or better than girls on Piaget Type Tasks at respective age levels.

The third hypothesis, i.e., the measures of intelligence (both verbal and non-verbal) correlate significantly with the variables of adolescent thought, stands tested in the light of the results presented in Table 23. The values of 20 coefficients of correlation, computed between the measures of verbal intelligence and non-verbal intelligence on one side and the ten variables of adolescent thought on the other side were found to be ranging between .402 and .560 which were all significant at .01 level.

A close view of the results presented in Table 23 reveals that the five measures of academic achievement in Mathematics, Science, English, Punjabi and Hindi were found to be correlated significantly (.01 level) with all the variables of adolescent thought except the variable of ratio and proportion (RAP), and the variable of space visualisation (V) was found to be correlated significantly only with the measure of academic achievement in Mathematics. Thus, out of the 50 coefficients of correlation, 40 coefficients of correlation were found to be significant at .01 level and one coefficient of correlation was significant at .05 level. On the basis of these results it has been concluded that there exists a

TABLE 21

COEFFICIENTS OF CORRELATION BETWEEN THE
PERFORMANCE ON PIAGET TYPE TASKS
AND OTHER VARIABLES

-----> PIAGET TYPE TASKS

OTHER VARS.	CL	GOT	GAA	PAC	RAP	FPO	ICI	STH	SV	GEP
IV	.385 ***	.456 ***	.538 ***	.535 ***	.389 **	.429 ***	.560 ***	.501 ***	.300 ***	.202 ***
INV	.403 ***	.416 ***	.520 ***	.462 ***	.287 ***	.323 ***	.487 ***	.267 ***	.334 ***	.235 ***
SR	.138 ***	.200 ***	.217 ***	.241 ***	.120 ***	.122 ***	.229 ***	.141 ***	.156 ***	.108 ***
RA	.202 ***	.367 ***	.417 ***	.454 ***	.318 ***	.362 ***	.435 ***	.426 ***	.257 ***	.128 ***
AAM	.154 ***	.093 ***	.187 ***	.141 ***	.057 ***	.136 ***	.159 ***	.104 ***	.094 ***	.110 ***
AAS	.147 ***	.117 ***	.190 ***	.150 ***	.039 ***	.170 ***	.146 ***	.143 ***	.056 ***	.111 ***
AAE	.096 ***	.121 ***	.149 ***	.131 ***	.048 ***	.138 ***	.134 ***	.132 ***	.059 ***	.094 ***
AAP	.091 ***	.152 ***	.152 ***	.103 ***	-.016 ***	.143 ***	.109 ***	.110 ***	.022 ***	.085 ***
AAH	.118 ***	.133 ***	.168 ***	.129 ***	.007 ***	.134 ***	.119 ***	.100 ***	-.002 ***	.075 ***
ADJ	.179 ***	.122 ***	.155 ***	.180 ***	.130 ***	.145 ***	.111 ***	.150 ***	.127 ***	.135 ***
A	.041 ***	.096 ***	.093 ***	.135 ***	.057 ***	.077 ***	.098 ***	.135 ***	.059 ***	.024 ***
B	.057 ***	.116 ***	.136 ***	.174 ***	.117 ***	.099 ***	.105 ***	.145 ***	.095 ***	.060 ***
C	.098 ***	.141 ***	.159 ***	.137 ***	.147 ***	.131 ***	.183 ***	.175 ***	.063 ***	.090 ***
D	-.021 ***	-.048 ***	-.101 ***	-.065 ***	-.020 ***	-.082 ***	-.070 ***	-.065 ***	-.023 ***	-.073 ***
E	-.083 ***	-.189 ***	-.132 ***	-.137 ***	-.020 ***	-.064 ***	-.123 ***	-.088 ***	-.021 ***	-.092 ***
F	.030 ***	.059 ***	.025 ***	.073 ***	.014 ***	.037 ***	.051 ***	.005 ***	.057 ***	-.003 ***
G	.148 ***	.163 ***	.218 ***	.232 ***	.140 ***	.152 ***	.213 ***	.177 ***	.124 ***	.072 ***
H	.096 ***	.185 ***	.169 ***	.209 ***	.129 ***	.102 ***	.154 ***	.172 ***	.043 ***	.056 ***
I	-.072 ***	.021 ***	.007 ***	-.004 ***	-.024 ***	.042 ***	.039 ***	.055 ***	-.074 ***	-.041 ***
J	.009 ***	-.048 ***	-.025 ***	-.002 ***	-.012 ***	.030 ***	-.028 ***	-.008 ***	-.007 ***	-.069 ***
O ₁	-.082 ***	-.176 ***	-.185 ***	-.183 ***	-.063 ***	-.170 ***	-.177 ***	-.181 ***	-.067 ***	-.096 ***
Q ₂	.018 ***	.048 ***	.056 ***	.032 ***	.083 ***	.053 ***	.088 ***	.084 ***	.052 ***	.018 ***
Q ₃	.097 ***	.180 ***	.206 ***	.178 ***	.099 ***	.171 ***	.204 ***	.167 ***	.070 ***	.086 ***
Q ₄	-.139 ***	-.180 ***	-.183 ***	-.163 ***	-.105 ***	-.130 ***	-.129 ***	-.135 ***	-.067 ***	-.104 ***

*** Significant at .05 level

** Significant at .01 level

significant relationship between the measures of academic achievement and the variables of adolescent thought which leads to the confirmation of the fourth hypothesis.

It is evident from the results presented in Table 23 that the coefficients of correlation computed between the measures of reasoning ability (RI) and space relations (SR) on the one hand and the measures of adolescent thought on the other hand were all significant at .01 level and were found to be ranging between .103 and .454. Thus, the fifth hypothesis, i.e., the measures of reasoning ability and space relations yield significant correlations with the various measures of adolescent thought, has been empirically verified.

All the coefficients of correlation between the measure of adjustment (AN) and the variables of adolescent thought were found to be significant at .01 level and their values were ranging between .111 and .180 (vide Table 23). It has been concluded that the measure of adjustment is significantly related to the variables of adolescent thought. The sixth hypothesis, thus, stands tested.

The seventh hypothesis, i.e., the measures of different traits of personality exhibit significant correlations with measures of the dimensions of adolescent thought, has been confirmed partially as some of the personality traits exhibited a consistent picture of correlations with all the measures of the dimensions of adolescent thought while the others were found to be correlated only with the measures of some specific dimensions of adolescent thought (vide Table 23). Thus, all the measures of

the fourteen traits of personality cannot be taken as correlates of each and every dimension of adolescent thought because the position of the correlates has been found to be changing from one dimension to the other. The description of the correlations of the personality traits* with the dimensions of adolescent thought have been given below with reference to the results presented in Table 23.

i) The reserved-outgoing (1) trait of personality was found to be having positive correlation with the dimensions of adolescent thought : grouping of thought (VII), generalisations to arithmetical and algebraic symbols (XII), permutations and combinations (XIV), interpretation and coordination of information (XVI), stating and testing hypotheses (XVII), which were significant at .01 level and with the dimension of formulation of probing questions (XX) which was significant at .05 level.

ii) The concrete-abstract thinking (2) trait of personality showed consistently positive correlation with all the dimensions of adolescent thought except the dimension of classification (X). All the coefficients of correlation were found to be significant at .01 level leaving aside the case of the dimension of grasping the essence of the problem (XIII) where it was significant at .05 level.

iii) The trait of emotional instability - stability (3) also showed positive correlation with all the ten dimensions of adolescent thought. The coefficients of correlation were all significant at .01 level except that with the dimension of space visualisation (XV) which was significant at .05 level.

* Low-High scores

iv) The phlegmatic-excitabile (P) trait of personality was found having negative correlation with all the measures of the dimensions of adolescent thought, however, all the coefficients are not significant. The coefficients of correlation between the phlegmatic-excitabile trait and the dimensions of generalization to arithmetical and algebraic symbols (GAS) and formulation of probing questions (FPQ) were found to be significant at .01 level and those of the dimensions of permutations and combinations (PAC), interpretation and coordination of information (ICX), stating and testing hypotheses (STH) and grasping the essence of the problem (GEP) were significant at .05 level.

v) The obedient - assertive trait (A) also showed a trend of negative relationship with all the dimensions of adolescent thought. The dimensions of classification (CL), grouping of thought (GT), generalization to arithmetical and algebraic symbols (GAS), permutations and combinations (PAC), interpretation and coordination of information (ICX), stating and testing hypotheses (STH) and grasping the essence of the problem (GEP) were found to be having negative correlations significant at .01 level while the dimension of formulation of probing questions (FPQ) was having a negative correlation significant at .05 level.

vi) No significant correlation was found between the serious-headedness (F) trait of personality and the dimensions of adolescent thought except that of the dimension of permutations and combinations (PAC) which was having positive correlation significant at .05 level.

vii) The expedient-conscientious (G) trait of personality showed consistently positive correlation with all the dimensions of adolescent thought. All the coefficients of correlation were significant at .01 level except the one that of the dimension of grasping the essence of the problem (G2) which was significant at .05 level.

viii) The shy-adventurous (I) trait of personality was also found to be having consistently positive correlation with all the dimensions of adolescent thought except the two, those of space visualization (IV) and grasping the essence of the problem (G2). All the above mentioned coefficients of correlation were significant at .01 level.

ix) The toughminded-tenderminded (J) trait of personality did not exhibit significant relationship with the measures of the dimensions of adolescent thought except in the case of two dimensions, i.e., classification (G5) and space visualization (IV). The coefficients of correlation of these two dimensions were negative and significant at .05 level.

x) The zealous-circumspect (J) trait of personality was found to be having a significant correlation with only one dimension of adolescent thought, i.e., grasping the essence of the problem (G2). The coefficient of correlation was negative and significant at .05 level.

xi) The secure-insecure (J) trait of personality showed consistently negative correlation with all the dimensions of the

adolescent thought. All the coefficients of correlation were significant at .01 level except in the case of the dimensions of ratio and proportion (RP) and space visualization (SV) which were significant at .05 level.

xii) The group dependent - self sufficient (η_2) trait of personality did not show any correlation with the dimensions of adolescent thought except in the case of ratio and proportion (RP), interpretation and coordination of information (ICI) and stating and testing hypotheses (TH). The coefficients of correlation were found to be positive and significant at .01 level.

xiii) The uncontrolled-self disciplined (η_3) trait of personality was found to be having consistently positive relationship with the measures of all the dimensions of adolescent thought. All the coefficients of correlation were significant at .01 level except one that of the space visualization (V) which was significant at .05 level.

xiv) The relaxed-tense (η_4) trait of personality showed negative relationship with the measures of all the dimensions of adolescent thought. All the coefficients of correlation were found to be significant at .01 level except in the case of space visualization (V) which was significant at .05 level.

Interpretations and Discussion

It may be recalled that the exploration of the relationship of the development of the different dimensions of adolescent thought with the variables of age, sex, intelligence, academic achievement, reasoning ability, space relations, adjustment and other personality traits, and the determination of the structural structure of adolescent thought, were the major objectives of this study.

The results of the study show that the development of the various dimensions of adolescent thought is contingent upon the age during the formal-operational period (vide Table 10). There is a clear evidence of a regular growth of the various dimensions of adolescent thought through 11⁺, 12⁺, 13⁺, 14⁺ and 15⁺ years of age. However, in the case of the dimensions of ratio and proportion (10), space visualization (7) and grasping the essence of the problem (6) a dip has been noticed in the process of the growth of these dimensions (Fig. 8), which may be characterized as the transitional period in the settlement of the new strategies and concepts in the minds of the adolescents (Gaiya and Gandhi, 1979). The relationship of the development of adolescent thought with the age does not need a special explanation as it is well understood that as the age increases, the experiences of the adolescents accumulate, and increase the chances to reach a state of equilibration through the processes of assimilation and accommodation carried out from time to time. The incompleteness of the development of the dimensions of adolescent thought even upto the age of 15⁺ years

indicates the under-functioning of the adolescents at formal level and thus the possibility of the extension of the development period to a higher age level.

The performance of boys has been found to be either equal to or better than girls on almost all tasks at respective age levels as well as for the combined groups, and in no case girls have shown better performance than boys (ages 12 to 16). It is very difficult to say whether the superiority of the boys over the girls with respect to their performance on almost all tasks is genetical. However, it can be explained well on the basis of social and cultural set up from which the sample has been drawn. In the rural culture of Punjab, girls do not get sufficient opportunities, particularly, during adolescence to interact with the world both scientific and social. They usually remain restricted to a sort of domestic environment. Thus, the development of formal thought, which takes place only through an interaction with a nexus of supportive and stimulating factors in the environment, is hindered in the case of the girls.

The development of the various dimensions of adolescent thought has been found to be co-axial with the measures of intelligence both verbal and non-verbal, reasoning ability and space relations (vide Table 43) which may be described as the allied abilities. The adjustment of an individual also plays a significant role in the development of adolescent thought. If viewed psychologically it seems quite meaningful as adjustment is indispensable for mental health which ensures maximum

effectiveness of mental abilities. A well adjusted individual brings out a balance among his intellectual, emotional and physiological satisfactions. Moreover, he has a clear insight into his own abilities and limitations. Thus, a well adjusted individual lives an orderly life in which the necessary functions of living are so regularized that a good deal of energy is made available for the organization of the functioning of the various aspects of cognitive, affective and social life of the individual. As a result he marches forward toward the

actualization of his potential. The measures of the academic achievement in the school subjects, such as, Mathematics, Science, English, Punjabi and Hindi have been found to be having significant bearings on the development of adolescent thought. As the contents of the subjects like Mathematics and Science are very much in harmony with and provide good exercises for the development of the various dimensions of adolescent thought, and the languages like English, Punjabi and Hindi serve as vehicles of thought, the students having got mastery over these subjects do come across a lot of manipulative and imaginative type of experiences which are the necessary and desirable pre-requisites of the development of adolescent thought. Moreover, the various concepts developed among the students during the studies of these subjects may also be helpful in the development of adolescent thought.

The personality traits investigated in the present research fall into the category of the structural factors of personality which comprise temperamental qualities, constitutional predisposition and basic behavioural patterns. These factors

are biologically based and constitutionally determined and as such, they are relatively stable and lasting characteristics of personality (Chalival, 1977). Ten factors of personality, out of the fourteen factors of personality studied (FTF), are found to be correlated significantly with six or more than six dimensions of the ten variables of adolescent thought investigated. Thus, the traits like outgoing tendencies, abstract thinking, emotional stability, phlegm, obedience, conscientiousness, adventurousness, feeling of security, self-discipline and relaxation correspond with the development of adolescent thought. The examination of these traits of personality reveals that they are not quite independent of each other, rather they are liable to be grouped together into certain clusters of functionally related traits. Thus, the ten traits of personality related with the development of adolescent thought may be grouped into two main categories. The first category consisting of emotional stability, phlegm, obedience, feeling of security, self-discipline and relaxation traits, corresponds to a specific type of personality temperament of an individual who is prone to be serious-minded, calm, cool, contented, disciplined and obedient. This type of person conserves his energy and puts it in the direction he desires. The second category, including traits of outgoing-tendencies, adventurousness, conscientiousness and abstract thinking, is associated with a specific type of behaviour. The person having this sort of personality is self-initiating, believes in adventurous activities and has markedly developed abstract thinking. Thus, the energy conserved by him at one stage is utilized for the activities which germinate the development of adolescent thought.

CHAPTER VI

Analysis of Mathematical Structure
Underlying the Adolescent Thought

CHAPTER VI

Analysis of Mathematical Structure
Underlying the Adolescent Thought

Introduction

Factor analysis, like all other statistics, is a branch of applied mathematics which is used largely as a tool to provide a mathematical model to explain the underlying behaviour of the data (Jarrow, 1960). It involves the analysis of a large battery of tests in order to identify a few common factors. Thus, the tests which best measure these factors may be considered direct measures of the 'factors of mind.' The principal concern of factor analysis, therefore, is the resolution of a set of variables linearly in terms of a small number of factors. Secondly, the individual factor loadings of the various tests included in a battery on the factors provide mathematical information about the behavioural composition of the tests and are, thus, the source of direct and concrete evidence of the tests' validity.

It has been accepted by now that a given matrix of correlations can be factorized in a number of ways. Thus the choice of a method out of the large number of equally accurate methods available, rests with the investigator which can wisely be made keeping in view statistical simplicity and psychological meaningfulness of the factor solution. A statistically sound approach would be to represent the original set of variables in terms of a small number of factors, determined in sequence so

that at each successive stage the factor would account for a maximum of the variance. This type of solution is possible by the Principal Axes Method of factor analysis which has been employed in the present study.

Some Background Studies

Abramowitz (1975) conducted a study on a group of 33 seventh grade students of 12-13 years. A revised version of Varplus's proportionality test organized into test booklets containing six tasks was administered to the subjects in group settings. All the tests that had to do with the handling of fractions were loaded on factor I which accounted for 33.7% of the variance; the average proportionality score and ability were loaded on factor II which accounted for 16.6% of the variance; the size contrast, the ratio contrast and correlative test of inverse relations were loaded on factor III which accounted for 19.7 % of the variance. This depicted the nature of the components of proportional thought as the skill tests of facility with fractions load on a different factor than tasks involving proportionality.

Bart (1971) administered four tasks of formal operations: Pendulum Task, Conservation of Motion on a Horizontal Plane, Equilibrium in the Balance and Projection of Shadows, to a group of 90 students of above average scholastic ability and belonging to three age levels of 13, 15 and 19 years. Also a test of verbal intelligence and the tests of formal reasoning in biology, history and literature were given. Factor analysis of these

eight measures was done. The eight measures were found to be having bi-factor structure with a large substantial first factor of formal operations and a second factor related to content which distinguished tasks from tests. Formal thought was found to have a substantial verbal intelligence component as well as non-verbal intelligence component.

Mullis, et al (1962) reported an inter task consistency of Piaget's levels representing a general Piagetian factor independent of any general intelligence factor entering into the Binet scale.

Guerin (1975) administered Piaget's test of logical operations to 896 students from four different environments to get the measures of attainment of seven logical operations - classification, seriation, multiplication, compensation, proportional thinking, probability and correlational thinking - to associate with the final two stages of Piaget's cognitive development. The structure of the logical operations described above consisted of two correlated factors, i.e., the concrete operational factor and the concrete and logical factor. The logical operation of compensation was related to both factors and represented a transition operation between the purely concrete operations and the purely formal operations.

Wattney (1975) found that Piagetian measures bore a modest positive degree of relationship to performance on traditional measures of intelligence, and thus, clarified that the two types of measures were neither totally distinct nor totally

identical. Both the measures were found to be contributing to a general intelligence factor. It also found that Piagetian factors were having a dominant concurrent association with the measures of school achievement, thus, pointed out the way to new and possibly more reliable and valid predictors of achievement.

Jovell and Lutterworth (1966) found a "central intellectual ability" underlying the performance on the tests of proportionality using the Principal Component Analysis.

Stephens, et al (1969) reported significant correlations of *ITC* verbal IQ, performance IQ and a full scale IQ with Piagetian tasks of reasoning and formal operations administered to the subjects of 6 to 12 years age. They have identified a general intelligence factor when the data were put to factor analysis. The fact that the Piagetian measures were covered by three factors, supported the notion of a multidimensional structure of intelligence in the Piagetian assessed areas as in the other traditional areas.

Vaidya (1975) investigating the growth of logical thinking during adolescence using seventeen problems and also the measures of intelligence, adjustment and immediate test reaction inventory, etc., on a sample of 200 students of grade VI to X found that unexpectedly the factor loading of intelligence on the first factor was insignificant. In all, the following ten factors were extracted:

1. Schematic Learning General
2. Adjustment
3. Problem Orientation

4. Censuring Problems
5. Symbolisation
6. Testing Hypotheses
7. Using Constant Differences
8. Aspect Character
9. Seeing the problem as a whole
10. Intelligence

Weeks (1973) administered three tests : Piaget's test of conservation of volume, Peul's story test to measure logical reasoning and Helmark's test of the understanding of correlation and the ability to use combinatorial analysis, to a sample of 190 seventh graders, 195 eighth graders and 175 ninth graders to measure the development of formal operations. Factor analysis yielded factors that could be reliably classified as either formal operations or verbal reasoning or numerical ability. The loadings of the items indicated a surprising mutual exclusivity of formal operations and verbal reasoning as operationally defined by the tests.

The background studies discussed above show that very few have taken sample of more than 200 subjects and have handled more than ten variables. Thus the attempts made by the various research workers were handicapped either from the sample viewpoint or with respect to the comprehensiveness of the study or both.

The Present Study

The major objective of this investigation was to make a comprehensive study of the mathematical structure of adolescent thought using ten Piaget Type Tasks and also taking the measures of the other variables such as intelligence (verbal and non-verbal), reasoning ability, space relations, academic

Considering the half matrix of intercorrelations of the ten Piaget Type Tasks presented in Table 24, it was found that all the 45 coefficients of correlation were positive and significant at .01 level. This showed that the performances on ten Piaget Type Tasks formed an interrelated measures of adolescent thought. The factorial structure was determined subjecting the above correlation matrix to factor analysis by Principal Axes Method. Results regarding the factorial structure of the tasks are presented in Table 25.

TABLE 25

ORIGINAL FACTOR MATRIX & THE FACTOR LOADING OF PIAGET TYPE TASKS ON THE ONLY SIGNIFICANT FACTOR AFTER ROTATION

Task No.	Task	Factor I
1	IL	.600
2	COF	.653
3	GA1	.755
4	PAC	.726
5	RIE	.465
6	TPP	.668
7	ICI	.705
8	W1	.689
9	IN	.574
10	GA2	.405
Sum of Squares		3.981
% of Total Variance		39.8
% of Common Variance		100.0

The results presented in Table 23 clearly demonstrate the unifactor structure of the measures of Piaget Type Tasks as only a single significant factor (having eigen value >1) could have been extracted through factor analysis. This single factor accounts for 39.8% of the total variance which indicates the uni-dimensionality of the formal thought running through all the tasks. The factor may be named, very conveniently, as Factor of Formal Thought. Since there is only one factor, rotation makes no sense. Secondly, the factor loadings of the various Piaget Type Tasks on the Factor I (only factor) may be taken as the indices of the factorial validity of the respective tasks which are all quite significant and high (ranging from .405 to .759). Thus, the eighth hypothesis stands verified and proved as the performances on Piaget Type Tasks form an inter-related measure of adolescent thought and exhibit a unifactor structure.

The ninth hypothesis of this study was stated as : the measures of intelligence, academic achievement, reasoning ability, space relations, adjustment and personality traits cluster in specific constellations with the measures of the dimensions of adolescent thought, explaining thereby the common factor variance. To test this hypothesis the data regarding all the 34 measures included in this study were put to factor analysis using Principal Axes Method. The various steps undertaken in this regard are described ahead. As mentioned earlier, thirty-four variables of different abilities with respect to each subject were investigated in the present study. The various

measures, in the same serial order as employed in the correlation matrix, are described below:

1. Intelligence verbal;
2. Intelligence non-verbal
3. Adjustment
- 4-17. Fourteen traits of personality (EPQ)
18. Space relations
19. Reasoning ability
- 20-29. Ten Measures of Piaget Type Tasks.
- 30-34. Academic achievement in five school subjects.

Correlation Matrix

A correlation matrix (34 x 34) presented in Table 26 was obtained using the above mentioned 34 measures for the whole sample (N=226) of the study. Considering the half correlation matrix, divided symmetrically by the diagonal, it was found on physical counting that it contained 561 coefficients of correlation out of which 426 were positive and the remaining 135 were negative. Out of the 426 positive coefficients of correlation 292 were found to be significant at .01 level, 317 significant at .05 level and 109 insignificant. Out of the 135 negative coefficients of correlation 53 were found to be significant at .01 level, 75 significant at .05 level and 60 insignificant. The magnitudes of the coefficients of correlation, irrespective of the signs, were found ranging from .001 to .597.

Obtaining the Factors

The correlation matrix (34 x 34) discussed above was subjected to factor analysis using the Principal Axes Method. Eight significant factors, having eigen values greater than one,

were extracted and retained for Varimax Rotation. All the computations were done through "ELITE - 1022 Computer" at Comptronics India, New Delhi, using 11-1 programme of factor analysis given in Statistical Package for the Social Sciences (SPSS). Results regarding the factor loadings of the 34 variables included in this study on the 8 significant factors are presented both in the case of Original Factors as well as Varimax Rotated Factors in Tables 27 and 28 respectively.

It is evident from the results presented in Table 27 that, in the case of Original Factors, the accumulated percentage of the total variance accounted for by the factors I through VIII has been found to be ranging from 19.3 to 49.1 while the percentage of common variance accounted for by the same factors is ranging from 39.3 to 100. The results presented in Table 28 clearly demonstrate that the accumulated percentage of the total variance accounted for by the factors I through VIII, in the case of Varimax Rotated Factors, has been ranging from 15.4 to 49.0 and the accumulated percentage of common variance accounted for by these factors range from 31.5 to 100.0.

Interpretation of Factors

Before presenting data for the interpretation of the factors, it is necessary to simplify the apparent factorial complexity regarding the various variables included in the study by ignoring the small sizes of the different factor loadings for pinpointing the attention on the significant factor loadings only whose substantial contributions could be

TABLE-27

137

ORIGINAL FACTOR MATRIX

FACTORS →

S NO	Variable	1	2	3	4	5	6	7	8	h ²	1-h ²
1	IV	-795	125	061	067	-201	024	041	092	706	294
2	INV	-690	100	-124	074	020	004	078	-136	532	468
3	ADJ	-309	046	031	-279	361	-095	156	-043	342	658
4	A	-224	061	385	-212	084	-010	136	150	295	705
5	B	-274	052	300	005	-264	-289	282	016	399	601
6	C	-307	126	472	-215	276	134	-111	-074	491	509
7	D	121	-026	119	459	-468	-233	125	-066	534	466
8	E	234	057	129	285	400	-172	-123	487	598	402
9	F	-114	-002	387	072	-094	-342	-094	-341	419	581
10	G	-376	076	371	-237	-035	-117	-093	105	376	624
11	H	-340	084	463	-273	082	072	168	-010	471	529
12	I	-003	-003	300	251	033	659	223	-116	648	352
13	J	041	015	239	347	162	-134	183	049	260	740
14	Q ₁	300	047	293	392	318	-154	173	-071	489	511
15	Q ₂	-101	121	351	217	-059	177	-563	-085	554	446
16	Q ₃	-350	067	426	-064	-156	-111	-252	-079	418	582
17	Q ₄	302	-022	167	584	091	188	038	-124	521	479
18	SR	-320	075	-102	-006	222	-017	448	-267	440	560
19	RA	-662	106	-054	052	-177	-046	150	222	560	440
20	CL	-544	077	-211	154	183	-172	-100	-158	467	533
21	GOT	-616	103	-100	019	-119	121	013	-228	482	518
22	GAA	-718	090	-133	101	036	058	-042	705	558	442
23	PAC	-695	149	-080	090	-012	-025	095	071	535	465
24	COR	-433	225	-010	088	007	-018	001	254	311	689
25	FPS	-596	066	-119	175	108	046	-142	154	463	537
26	ICI	680	135	-072	125	-074	168	-002	-047	537	463
27	STH	-623	134	-031	100	-041	199	-021	264	528	472
28	SV	-447	182	-181	229	219	-302	-124	-027	473	527
29	GEP	-337	014	-186	-030	240	-061	-239	-392	435	565
30	AAM	-347	-586	015	026	104	-034	040	-043	480	520
31	AAS	-335	-643	-017	041	091	100	-035	051	547	453
32	AAE	-332	-701	047	048	-020	005	-077	072	618	382
33	AAP	-305	-711	064	030	054	-044	033	014	609	391
34	AAH	-326	-680	077	008	-063	-051	-006	022	582	418
Sum of squares		6.550	2.492	1.786	1.504	1.164	1.116	1.060	1.004	6.676	17.324
% Total variance		19.3	7.3	5.3	4.4	3.4	3.3	3.1	3.0	49.1	50.9
% Common variance		39.3	14.9	10.7	9.0	7.0	6.7	6.4	6.0	100%	

12-12
V-58

ALL DECIMALS HAVE BEEN OMITTED IN FACTOR LOADINGS

TABLE 28
VARIMAX ROTATED FACTOR MATRICES
FACTORS →

S.NO.	Variable	1	2	3	4	5	6	7	8	9	10
1	IV	758	126	243	-134	181	312	-005	024	717	299
2	XIV	635	107	101	-115	073	043	263	-140	511	367
3	ADJ	153	053	357	012	-172	-117	316	-304	312	084
4	A	097	031	514	050	015	001	-122	-058	218	754
5	B	194	037	256	038	484	-084	-123	-45	400	300
6	C	110	002	622	045	-124	-134	191		490	310
7	D	-003	-010	-276	170	623	058	-164		534	277
8	E	-061	-057	-007	035	-271	-236	-210		590	377
9	F	-063	047	251	058	469	-060	306	146	410	587
10	G	212	054	523	-073	102	-144	-016	130	370	674
11	H	136	038	649	-032	064	122	-006	-080	470	530
12	I	036	018	099	107	-065	760	107	033	448	352
13	J	012	012	046	473	155	080	-015	-034	258	777
14	O ₁	-233	-089	015	521	116	128	386	-056	468	511
15	O ₂	095	-080	135	064	038	169	166	680		
16	O ₃	109	060	012	-070	268	-046	117	335		
17	O ₄	-152	-038	-259	462	102	424	023	160		
18	SR	247	011	102	044	044	154	260	-519	532	
19	RA	684	102	141	-093	135	-079	-132	-106	530	
20	CL	513	095	-030	047	-002	-140	415	-055	468	552
21	GOT	649	069	083	-259	114	149	255	-045	482	510
22	GAA	609	135	089	-085	-032	011	186	-013	359	441
23	PAC	697	074	141	-038	060	-030	082	-109	535	464
24	PAP	617	064	129	064	-033	-091	-090	-026	310	690
25	FPC	552	137	037	049	-131	-032	103	101	461	539
26	ICA	677	075	091	-137	040	157	139	015	537	463
27	STH	682	076	140	-053	-103	072	-114	074	528	472
28	SV	493	018	-047	206	008	-268	337	-009	473	527
29	GEP	225	083	002	-112	-099	-055	593	020	435	565
30	AAM	115	666	052	-003	004	003	114	-089	480	520
31	AAS	130	718	001	-033	-098	054	024	004	547	453
32	AAE	104	774	016	-041	015	-015	016	072	616	382
33	AAP	049	774	050	010	034	-008	030	-051	509	391
34	AAH	076	747	054	-067	102	-026	-009	007	592	418
Sum of squares		5248	2863	2159	1495	1247	1110	1306	1173	16661	17339
% Total variance		15.4	8.4	6.4	4.4	3.7	3.4	3.8	3.5	49.0	51.0
% Common variance		31.5	17.2	13.0	9.0	7.5	7.0	7.8	7.0	100.0	

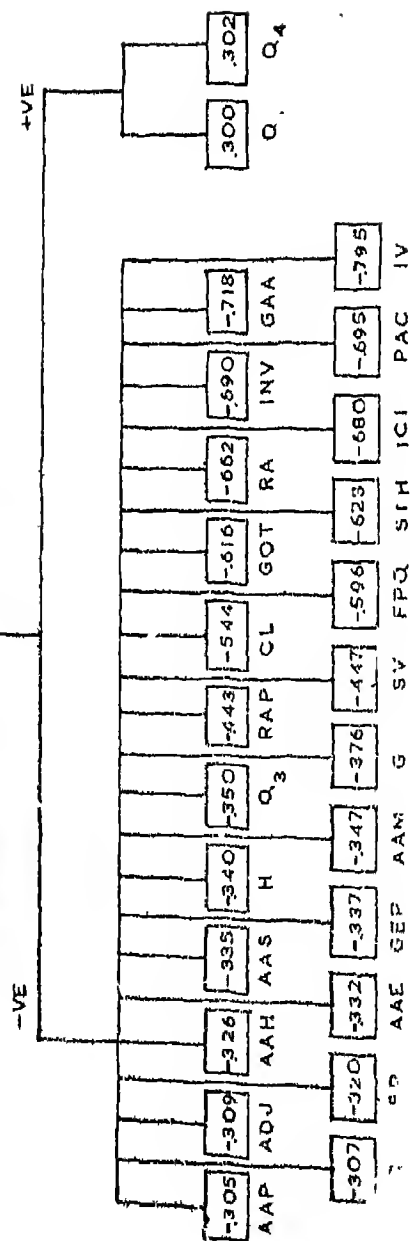
100%

ALL DECIMALS HAVE BEEN OMITTED IN FACTOR LOADINGS

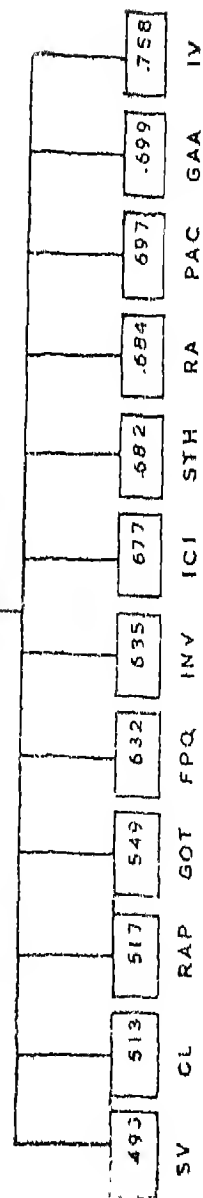
All loadings below $\pm .30$ have not been shown

FIG- 9

ORIGINAL FACTOR I SIGNIFICANT LOADINGS



VARIMAX FACTOR I SIGNIFICANT LOADINGS



considered for the determination of the nature of the factors. In practice there is no uniform criterion for judging how much small is small. Benjamin Fraichter (1967) has suggested that values of factor loadings less than .20 are, generally speaking, insignificant and hence can be ignored. Anderson and Eaton have ignored values upto .30 for the purpose of factorial interpretations (Vaidya, 1975). In the present study also the factor loadings of the different variables on the various factors having values numerically less than .300 have been ignored while interpreting the factors.

Factor I

Significant loadings of the different variables on Factor I (both Original and Varimax Rotated) have been shown in Fig. 9. A minute inspection of the figure shows that, in the case of Original Factor I, 26 variables are having significant loadings on this factor. This factor seems to be bi-polar in nature as the factor loadings of 24 variables - ten variables of the dimensions of adolescent thought (TL, GOF, GAA, P12, RAP, FPO, ICI, STI, JV and GSP), five variables of academic achievement (AAH, AAC, AAI, AAP and AAM), two variables of intelligence (IV, IW), a variable of reasoning ability (RA), a variable of space relations (RI), a variable of adjustment (ADN) and four variables of personality traits, (G, G, I, and 23) - have been found to be negative and thus, they lie on the negative pole. The range of the significant loadings has been varying from -.305 to -.798. The highest loading being that of the variable of verbal intelligence. Other variables are given

in the ascending order starting from III (-.305) and ending up with IV (-.795). The variables of two personality traits (Q_1 , Q_4) lie on the positive pole having factor loadings .300 and .302 respectively. This illusionary bi-polarity is only due to the reverse direction of scoring of the Q_1 and Q_4 traits of personality.

It is quite clear that this factor has brought together the variables of adolescent thought, academic achievement, intelligence, reasoning ability, space relations, adjustment and some of the personality traits on one continuum (constellation) as it is running through all the variables. It accounts for 39.3 % of the common factor variance and 19.3 % of the total variance (vide Table 27).

Varimax rotated Factor I provides a very simple structure. Twelve variables are found to be having positive loadings on this factor which include nine measures of the dimensions of adolescent thought (CV, TL, NAR, GOR, PRQ, ECI, STI, PII, CIA), two variables of intelligence (IN, IV) and a variable of reasoning ability (RA). The loadings have been found to be varying from .403 to .758. The highest factor loading has been shared by the variable of verbal intelligence. The other variables are presented in the ascending order, starting with CV (.403) and ending with IV (.758). This factor has brought together the measures of dimensions of adolescent thought, reasoning ability and intelligence. It accounts for 31.5 % of the common factor variance and 15.4% of the total variance (vide Table 28). On the basis of the nature of the loadings of

ORIGINAL FACTOR
SIGNIFICANT LOADINGS

535	-643	-580	-70
AAW	AP S	AAH	AAE

[illegible]

666	718	747	774	804
AAH	AAS	AAH	AAE	AAE

the various variables in the case of the Varimax Rotated Factor I, it can be named as General Intellectual Factor of Adolescent Thought.

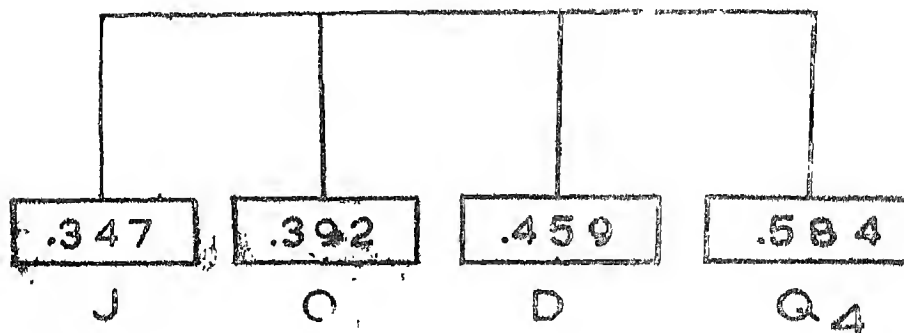
Factor II

Only the variables of academic achievement in school subjects (AM, AT, AI, AL, AW) are having significant loadings on Factor II, both in the case of Original Factor as well as Varimax Rotated Factor. The Factor loadings of the measures of academic achievement on Original Factor II are negative and ranging from $-.586$ to $-.711$. The measures of academic achievement put in the ascending order with respect to the size of factor loadings as presented in Fig. 10 are AM ($-.586$), AT ($-.643$), AI ($-.680$), AL ($-.701$) and AW ($-.711$). This factor accounts for 14.9 % of the common factor variance and 7.3 % of the total variance (vide Table 37).

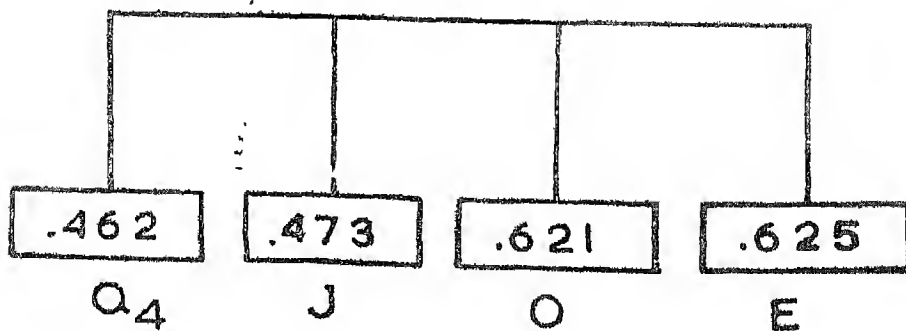
In the case of Varimax Rotated Factor II, the signs of the factor loadings become positive though the variables with significant loadings on this factor remain the same as in the case of Original factor. The range of the various factor loadings has been found to be varying from $.666$ to $.774$. The ascending order of the different variables with respect to the size of the factor loadings is AM ($.666$), AT ($.718$), AI ($.747$), AL ($.774$) and AW ($.774$). This factor accounts for 17.2 % of common factor variance and 8.4 % of total variance (vide Table 38). Since all the measures of academic achievement are having loadings almost of equal magnitude on Varimax Rotated Factor II, it has been

FIG.- 12

ORIGINAL FACTOR IV
SIGNIFICANT LOADINGS



VARIMAX FACTOR IV
SIGNIFICANT LOADINGS



named as Factor of Academic Achievement instead of naming this factor on the basis of a single measure of academic achievement.

Factor III

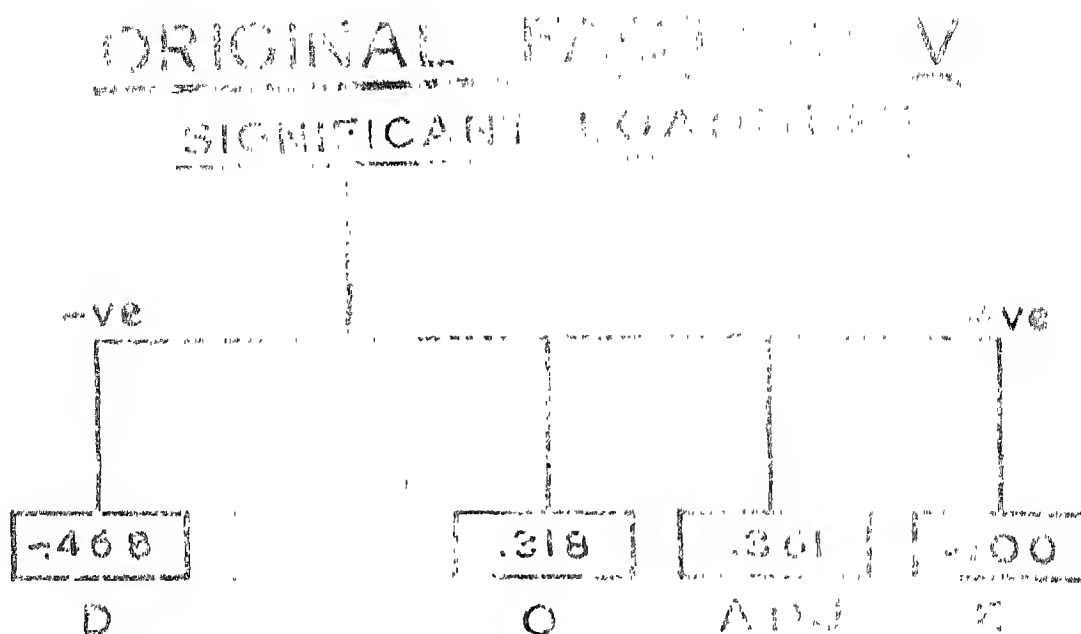
The results presented in Fig. 11 demonstrate that the measures of personality traits $X_1, I_1, Y_2, G_1, P_1, Y_3, C$ and I have significant positive factor loadings on Original Factor III. The highest loading being that of I (.483) and the lowest of X (.300). The loadings of the other traits are in the same order as they are given above. This factor accounts for 10.7 % of common factor variance and 5.3 % of total variance (vide Table 27).

In the case of Varimax Rotated Factor III only the measures of X (.357), Y_2 (.418), I (.514), G (.523), X (.622) and Y (.649) have significant loadings. It accounts for 13.0 % of common factor variance and 6.4 % of total variance (vide Table 28). This is clearly a group factor of personality. On the basis of the nature of the various traits of personality having significant factor loadings on Varimax Rotated Factor III, it can be named as Adjustment Factor.

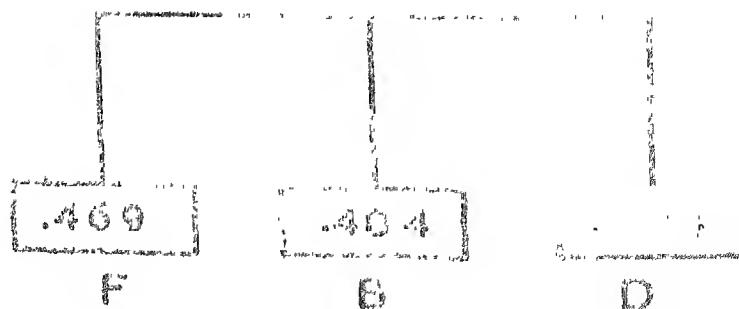
Factor IV

The factor loadings of personality traits X, Y, D and Y_4 are found to be significant on Original Factor IV (Fig. 12). The order of the size of the loadings is X (.347), Y (.392), D (.459) and Y_4 (.594). This factor accounts for 9.0 % of common factor variance and 4.4 % of total variance (vide Table 27).

FIGURE 13



VARIMAX FACTOR V
SIGNIFICANT LOADINGS



In the case of Varimax Rotated Factor IV, there is a change of only one variable, i.e., instead of measure D, the measure C has significant loading on this factor, other variables remaining the same as in the case of Original Factor. However, the sizes of loadings as well as the order of variables with respect to the size of factor loadings get changed. The new order is $-I_4(.463)$, J(.473), Q (.521) and L(.625). This factor accounts for 9.0 % of common factor variance and 4.4 % of total variance (vide Table 20). Keeping in view the comprehensive nature of the traits of personality having significant factor loadings, i.e., Q_4 (relaxed-tense), J(nestful-circumspect), Q (secure-insecure) and L(obedient-assertive), this factor can be named as Behavioural Factor.

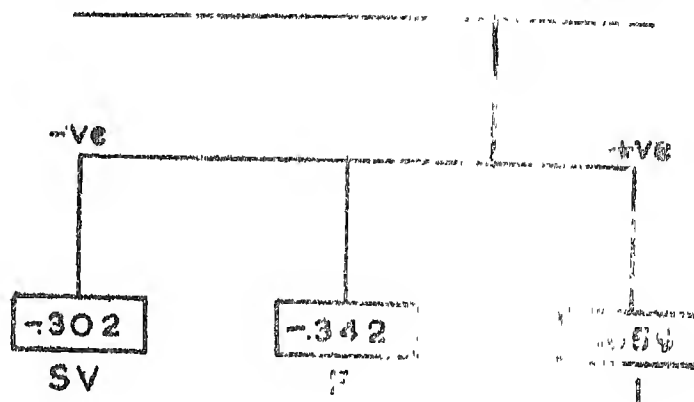
Factor V

Original Factor V shows bi-polar characteristics as the factor loading of the personality trait D(phlegmatic-excitabile) is negative while the factor loadings of the variables : Q (secure-insecure), ADJ (adjustment) and L(obedient-assertive) are all positive as given in Fig. 13. The highest loading has been shared by D(-.463). This factor accounts for 7.0% of common factor variance and 3.4 % of total variance (vide Table 27).

The personality traits, F,B and D are having significant positive factor loadings in the case of Varimax Rotated Factor V. The sequence of personality variables with respect to the size of factor loadings is F(.469), B(.434) and D(.623). This factor accounts for 7.5% of common factor variance and 3.7 % of total

FIG.- 14

ORIGINAL FACTOR VI
SIGNIFICANT LOADINGS



VARIMAX FACTOR VI
SIGNIFICANT LOADINGS

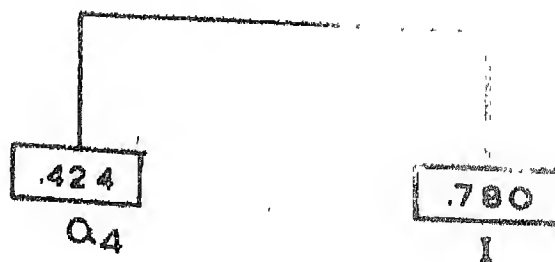
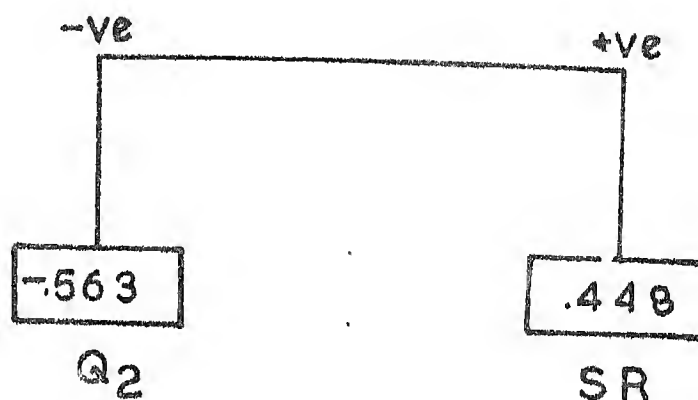
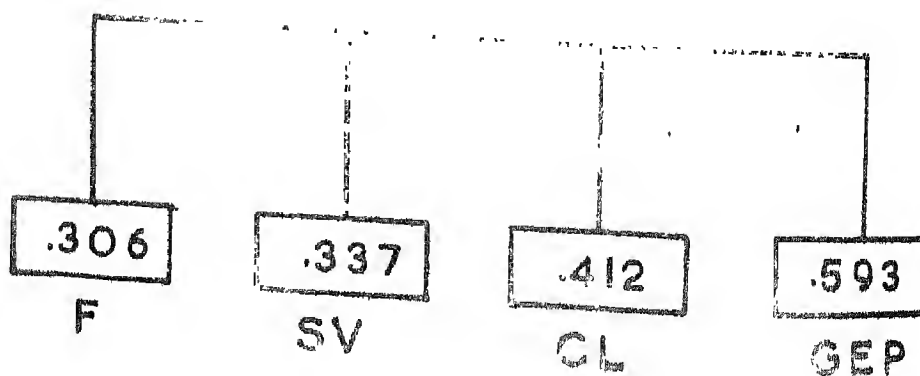


FIG. 15.

ORIGINAL FACTOR VII
SIGNIFICANT LOADINGS



VARIMAX FACTOR VII
SIGNIFICANT LOADINGS



variance (vide Table 23). In view of the characteristics of the personality traits -F(sober-headless), B(concrete-abstract thinking) and C(phlegmatic-excitabile) - having significant factor loadings, this factor may be named as Emotional Factor.

Factor VI

The results presented in Fig. 14 show that Original Factor VI is bi-polar in nature as the factor loadings of the variables S/(-.302) and P(-.342) are negative while that of the measure of I(.659) is positive. The highest loading is shared by I(toughminded-tenderminded). This factor accounts for 6.7 % of common factor variance and 3.3 % of total variance (vide Table 27).

In the case of Varimax Rotated Factor VI only two variables Q_4 (.424) and I(.730) are having significant factor loadings. It accounts for 7.0 % of common factor variance and 3.4 % of total variance (vide Table 23). On the basis of the adjectives used to describe the two traits of personality, i.e. Q_4 (relaxed-tense) and I(toughminded-tenderminded) having significant factor loadings on Varimax Rotated Factor VI, it can be named as Temperamental Factor.

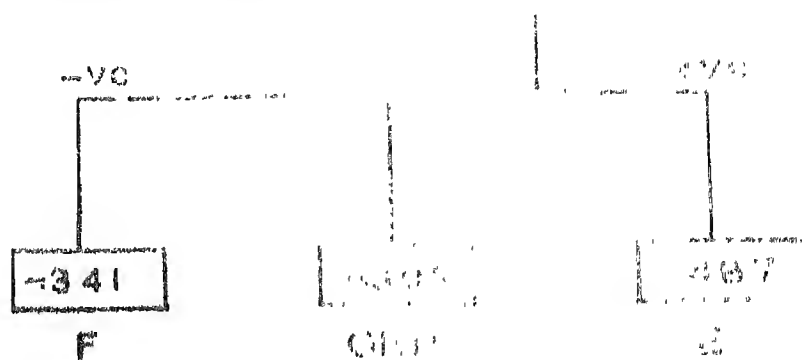
Factor VII

Original Factor VII also shows bi-polarity as out of the two variables having significant factor loadings on this factor, one (Q_2) has negative (-.563) loading while the other (CN) has positive (.493) factor loading (Fig. 15). It accounts for 6.4% of common factor variance and 3.1 % of total variance (vide Table 27).

FIG. 16

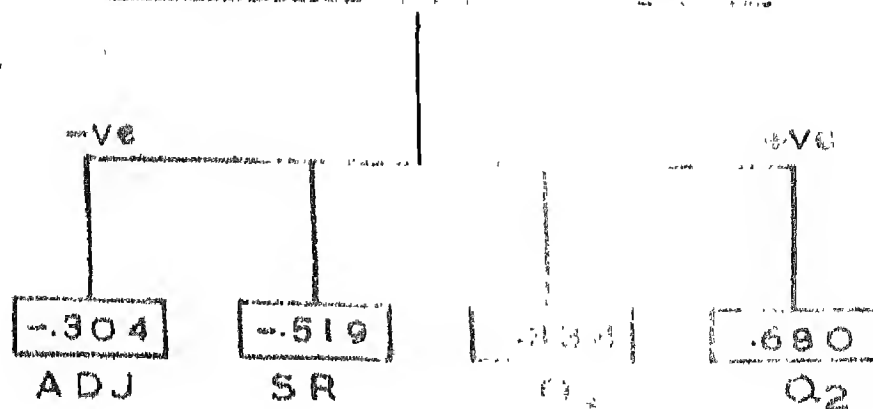
ORIGINAL FACTOR VII

SIGNIFICANT LOADINGS



VARIMAX FACTOR VIII

SIGNIFICANT LOADINGS



Varimax Rotated Factor VII has the significant loadings of the variables $F(.306)$, $IV(.337)$, $CL(.412)$ and $GP(.593)$. It accounts for 7.8% of common factor variance and 3.8% of total variance (vide Table 23). Three out of the four variables having significant loadings on this factor are the measures of the dimensions of adolescent thought. Hence, on the basis of the nature of the variables having high factor loadings, this factor may be named as Group Factor of Adolescent Thought.

Factor VIII

The results presented in Fig. 16 show that Original Factor VIII is bi-polar in nature. The variables $F(-.341)$ and $GP(-.393)$ have negative factor loadings while the measure of $A(.467)$ has positive factor loading. This factor accounts for 6.0% of common factor variance and 3.0 % of total variance (vide Table 27).

Varimax Rotated Factor VIII also shows bi-polarity. The factor loadings of the measures $GP(-.304)$ and $A(-.319)$ are negative and those of $Q_3(.334)$ and $Q_2(.680)$ are positive. It accounts for 7.0 % of total variance (vide Table 28). It is a group factor of a complex nature but on the basis of the nature of the variables having highest positive loadings, i.e., Q_2 (group dependent-self sufficient) and Q_3 (uncontrolled-self-controlled), it can be named as Social Factor.

The review of the descriptions of eight factors both original and varimax, clearly demonstrates that in the light of these results the ninth hypothesis of the study (the measures of intelligence, academic achievement, reasoning ability, space

relations, adjustment and personality cluster in specific constellations with the measures of the dimensions of adolescent thought explaining thereby the common factor variance) stands verified and approved.

The Current Picture of the Structure of Adolescent Thought

The factorial structure identified through this study consists of eight orthogonal factors extracted by Principal Axes Method. The factors are : General Intellectual Factor of Adolescent Thought, Academic Achievement Factor, Adjustment Factor, Behavioural Factor, Emotional Factor, Temperamental Factor, Group Factor of Adolescent Thought and Social Factor. The distinguishing characteristic of the structure of adolescent thought identified in the present study is that it consists of, in addition to intellectual and academic factors, a good number of personality factors. None of these factors could be identified in the earlier studies as they had not taken into consideration this hot cognition while studying the formal thought. Secondly, Piaget Type Tasks included in this study were administered as paper-pencil tests and thus, the attempt may be characterized as psychometric rather than clinical. Some other research workers like Longoot (1955), Raven (1973), Thayer (1979), Staver and Gabel (1979) and Tisher (1971) have also attempted to develop paper-pencil tests for investigating the adolescent thought and have extracted factors using different tasks or tests, populations and techniques of analysis. The

results of these studies indicate that the formal thought which appears during adolescence is being scratched factorially as majority of the studies are handicapped both in terms of adequate samples as well as of the dimensions of the adolescent thought investigated. Most of the studies are contented with one or two factors only as none of them has gone further than three factors except Bandhu, Vaidya and Vaidya & Misra.

It is not possible and logical even to compare the findings of the various studies which have attempted to analyse the formal thought mathematically. The reason being that the findings come from different samples using various tests and techniques. Even then for the sake of clarity, structurally speaking, if the different results are put side by side the picture that emerges is as follows:

No.	Factor	Psychological Interpretations	Authors
1	2	3	4
1.	First Factor	i. General Intellectual Factor ii. Schematic Learning General iii. General Adjustment iv. Formal Operational Thought v. Attainment Factor vi. Algebraic Aptitude	Inab (1964), Beard (1957), De Lencos (1969), Mac Arthur (1962), Peel (1953), Bandhu (1980), Stever and Gabel (1979), Tuddenham (1970). Bart (1971), Lawson (1975), Vaidya (1975) Vaidya and Misra (1974) Abramowitz (1978), Shayer (1979) Vaidya (1964) Joshi (1970)

1	2	3	4
2. Second Factor	i. Piagetian Cognitive Development.	Staver and Gabel (1973)	
	ii. Seeing the Problem as a whole.	Vaidya and Misra (1974)	
	iii. Academic Achievement Factor.	Sandhu (1980)	
	iv. Adjustment	Vaidya (1975)	
	v. Practical Factor	Vaidya (1964)	
	vi. Symbolic Substitution	Joshi (1970)	
3. Third Factor	i. Piagetian Logical Operations Test.	Staver and Gabel (1973)	
	ii. Formulating Hypotheses	Vaidya and Misra (1974)	
	iii. Adjustment Factor	Sandhu (1980)	
	iv. Problem Orientation	Vaidya (1975)	
	v. Interest Factor	Vaidya (1964)	
4. Fourth Factor	i. Interest in Generating Difficult Problems	Vaidya and Misra (1974)	
	ii. Behavioural Factor	Sandhu (1980)	
	iii. Sensing Problems	Vaidya (1975)	
	iv. Adjustment Factor	Vaidya (1964)	
5. Fifth Factor	i. Newness of the Problem	Vaidya & Misra (1974)	
	ii. Emotional Factor	Sandhu (1980)	
	iii. Symbolisation	Vaidya (1975)	

6. Sixth Factor	i. Temperamental	Gandhu (1980)
	ii. Testing Hypotheses	Vaidya (1975)
7. Seventh Factor	i. Group Factor of Adolescent Thought	Gandhu (1980)
	ii. Being Constant Difference	Vaidya (1975)
8. Eighth Factor	i. Social Factor	Gandhu (1980)
	ii. Aspect Character	Vaidya (1975)
9. Ninth Factor	Seeing the Problem as a whole	Vaidya (1975)
10. Tenth Factor	Intelligence	Vaidya (1975)

Concluding Statement

In this frame of reference, it should not be forgotten that factor analysis is a highly mathematical technique as well as an advanced educational technology. The various factors failing to appear in a definite way should be further subjected to empirical testing by carrying out highly imaginative studies using factorially known tests as reference points. The various tests used should cover as many diverse populations meeting fairly well the intended objective criteria of reliability and validity. Similarly, the sizes of the samples should not invariably be less than four times the number of tests used. After having done this, the growth of factors should be clinically explored and the same be checked

empirically to test whether the findings come from different chips of different blocks or the different chips of the same block. Whatever be the nature of these findings, they are bound to alter the already proposed structures of intellect, a periodic table of intelligence like the one in chemistry, if it exists, may become available for the benefit of learning psychologists. Until then, as it is apparent, the studies undertaken simply reflect the possible structure of adolescent thought as imaginatively proposed by the Geneva School, using symbolic logic.

THE NEW YORK

WAS EFFECT RECORDED : IN
OBSERVATION OF "COMMON INTEREST"

CHAPTER VII

Hump Effect encountered : An Observation of Second Interest

Jean Piaget, while working with Theophile Simon in Paris in 1930 to standardize a test, found that children of a particular age commit almost similar type of errors. Later on, keeping in view the similarities in the thinking processes at different age levels, he propounded the concept of the 'Stages of Development'. It is a fine coincidence that Vaidya (1975) too encountered 'hump' in the number of errors while investigating the errors committed by the students in solving some problems. He noticed a sudden increase in the errors with the increase in age during the formal-operational stage contrary to the expectations that the errors should go on diminishing with age. The number of errors, however, declined at subsequent age levels gradually to a minimum. He raised some pertinent issues about this phenomenon, i.e., Is it the case of an adolescent playing with figures thoughtlessly in the hope of being favoured with good luck? Is it a case of lack of seriousness on his part? Is it the case of being caught between the horns of a dilemma and getting mired? Is it the case of hot chase trying hard to choose in haphazard directions as if in the manner of closing in on the problem? Does it illustrate that mastery of a thought process is through a path uphill, thorny and often erratic? Does the adolescent regress as if on an adventurous Piagetian journey during which he is trying hard to educate

himself, thinking that the right path to concept development lies in flourishing on experimental failures or a problem solving situation in which either understanding suffers a dip or errors a hump? Is it a fact of rubbing his schemes of thought wrongly, especially when he has personal reservations about his self acquired knowledge in contrast to school learning which does not set right his firmly held self centred thoughts? These queries definitely needed further clarifications. The matter was referred to Prof. J.L. Bruner for comments and guidance about this elusive phenomenon. Bruner (1976) described that "The type of error that you refer to, which we speak of as growth error, is one in which a growing child tries out a new strategy although it is not well developed and uses it in place of an older one which has been working well. It is errors of this sort which suggest to me the venturesomeness of learning during this early period, the human beings are willing to shift to a less certain and more powerful strategy, before they have it under control, in preference to one which is safe, sound and dull." Later on, the phenomenon of 'hump effect' was also studied from the developmental aspect of thinking processes by Vaidya & Sandhu (1973)², where it has been found that 'hump effect' (dip in the real sense) appears when a thought process moves from a lower stage to a higher stage, particularly, during the transitional period, i.e., the period between any pair of the two succeeding stages. The data given in The Essential Piaget show that Piaget & Inhelder (1977), Lovell & Ogilvie

²See appendix (V).

(1977) and Alkind (1977) did encounter this phenomenon but missed referring to it in their studies.

In the present study also the 'hump effect' has been noticed on three dimensions of adolescent thought, i.e., ratio and proportion (RP), grasping the essence of the problem (EP) and space visualization (SV). The data regarding the mean scores and standard deviations of the different age groups with respect to the three dimensions of adolescent thought are presented in Table 30.

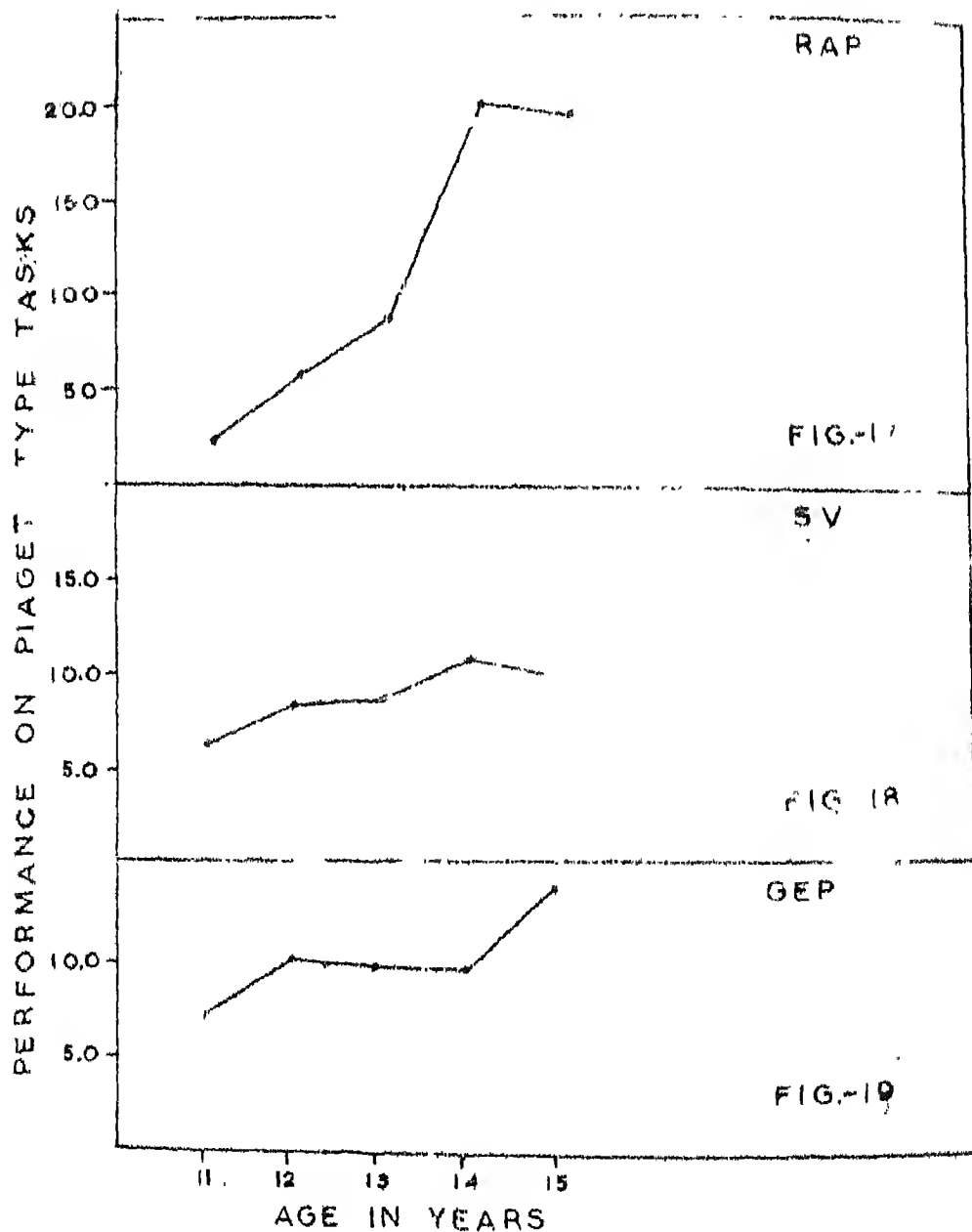
TABLE 30

MEAN SCORES AND STANDARD DEVIATIONS ON THE
DIFFERENCE OF RATIO AND PROPORTION, GRASPING THE
ESSENCE OF THE PROBLEM AND SPACE VISUALIZATION
AT DIFFERENT AGE LEVELS

Sr. No.	Dimensions of Adoles- cent thought	Mean Scores and Standard Deviations at Different Age Levels				
		11 ⁺	12 ⁺	13 ⁺	14 ⁺	15 ⁺
1.	RP	.832 (.558)	.898 (1.720)	.898 (1.832)	2.094 (2.614)	1.977 (2.560)
2.	EP	5.691 (3.869)	7.971 (4.734)	8.257 (4.693)	10.453 (4.543)	9.874 (4.709)
3.	SV	.679 (.965)	1.005 (1.080)	.960 (1.144)	.963 (.997)	1.394 (1.236)

Standard Deviations are given in brackets.

HUMP EFFECT AS OBSERVED REGARDING
THE PERFORMANCE ON DIFFERENT
PIAGET TYPE TASKS.



The phenomenon has been illustrated more clearly through the graphical presentation of the results given in Table 20. The figures 17, 18 and 19 demonstrate the 'hump effect' with respect to the dimensions of adolescent thought HA, SV and GEP respectively.

It is important to note here that the occurrence of hump or dip is not bound up with any particular age level in the case of the dimensions of adolescent thought described above, rather it occurs at different age levels depending upon the thought process under study. A similar trend has been noticed in the case of the children of lower age levels with respect to the concepts of concrete-operational stage in the studies of Piaget & Inhelder (1977), Lovell & Ogilvie (1977) and Alkind (1977). Thus, it can be said that 'hump effect' appears at all ages, the thought process being the determining factor, among pupils belonging to different intellectual levels when new schemes of thought are under development.

Though this phenomenon has been viewed quite comprehensively still there is a chance that sampling fluctuations might influence the number of errors as well as the performance on dimensions of adolescent thought at different age levels, since all the studies mentioned in this context are cross-sectional in nature. Hence, the phenomenon of 'hump effect' needs further verification through the longitudinal studies on the various aspects of the thought processes associated with the different age levels.

CHAPTER VIII

Summary and Conclusions

CHAPTER VIII

Summary and Conclusions

Introduction

Now-a-days the major interest of the psychologists and educationists is not only in understanding the individuals but in studying the general trend of the development and structure of the human mind. The scientific investigation of thinking processes and structure of human mind is gaining importance as the growth of highly logical mind has become one of the most important goals of the educational instruction in the modern society. Jean Piaget has contributed immensely to the whole field of psychology in general and to the modes of human thinking particularly. He speaks of qualitative changes in the underlying processes of thinking leading to the mental growth. He groups these qualitative changes into a succession of four global stages of development: the sensory-motor stage (birth to 2 years), the pre-operational stage (2 to 7 years), the concrete-operational stage (7 to 11 years) and the formal-operational stage (11 to 15 years). The important feature of Piaget's theory is that he is more interested in studying the cognitive structure of developing human mind than its function and content. Cognitive structure refers to the form or shape or pattern that cognition takes during each of Piaget's stages of mental development. The present study was undertaken with a view to investigate the structure of thought at formal-operational stage. At this stage, the refinements of adult

thought are acquired. The thinking processes become abstract and they no longer depend on the observed data. They can be carried out on hypothetical information. Thus, the formal thought is a generalised orientation, sometimes explicit and sometimes implicit, towards problem solving : an orientation towards organising data (combinatorial analysis), isolation and control of variables, the hypothetical and logical justification and proof. Piaget believes that intelligence reaches ultimate equilibrium at the formal-operational stage.

There is a sufficient research evidence that in many cases the subjects of the age group 11 to 15 years which we may call adolescents were found not developed to the formal-operational level. Since primitive structures form the basis of the more advanced-level structures, an inter-related criteria of covering the various dimensions of formal-operational thought alongwith a few dimensions of concrete-operational thought, were worked out and named as adolescent thought. Thus, adolescent thought shows a form of grouping : concrete operational and coordinating concrete-logical as described by (Aurein (1975) also.

It is true to say that in spite of large number of excellent works published on the affective, social and emotional domain of adolescents, little work has appeared in the psychological literature on the adolescent's thinking processes. However, thinking in general has been the subject of study from various standpoints by many philosophers and psychologists. There seems to be a scarcity of literature to which reference could be made regarding the mathematical analysis of the adolescent thought.

The effort has been made by this study to analyse the adolescent thought mathematically through factor analysis to identify its underlying structure, and to explore the relationship of the development of adolescent thought with the variables of age, sex, intelligence, academic achievement, reasoning ability, space relations, adjustment and other personality traits. The ten dimensions of adolescent thought investigated through the present study consisted of the measures of classification, grouping of thought, generalization to arithmetical and algebraic symbols, permutations and combinations, ratio and proportion, formulation of probing questions, interpretation and coordination of information, stating and testing hypotheses, space visualization and grasping the essence of the problem. A paper-pencil test of Piaget Type Tasks based on the above dimensions of adolescent thought was developed by the investigator for group administration.

Hypotheses

The study was undertaken to test the following hypotheses:

1. Does the performance on Piaget Type tasks increase with age during the formal-operational period?
2. Whether boys and girls perform equally well on Piaget Type Tasks?
3. The measures of intelligence, both verbal and non-verbal, correlate significantly with the measures of the dimensions of adolescent thought.

4. There exists a significant relationship between the measures of academic achievement and the variables of the dimensions of adolescent thought.
5. The measures of reasoning ability and space relations yield a significant correlation with the various measures of adolescent thought.
6. The measure of adjustment is significantly related to the performance on Piaget Type Tasks.
7. The measures of personality exhibit significant relationship with the measures of the dimensions of adolescent thought.
8. The performances on Piaget Type Tasks form an interrelated measure, of the adolescent thought and exhibit a unifactor structure.
9. The measures of intelligence, academic achievement, reasoning ability, space relations, adjustment and other personality traits cluster in specific constellations with the measures of the dimensions of adolescent thought explaining thereby the common factor variance.

Sample

A sample of 986 students (503 boys and 483 girls) was drawn randomly among the students of twelve high schools of the rural areas in Punjab taking almost equal number of boys and girls of the age groups of 11⁺, 12⁺, 13⁺, 14⁺ and 15⁺ and studying in grades VI, VII, VIII, IX and X respectively.

Tools

The data were collected using the following tools:

1. Test of Piaget Type Tasks (Investigator).

2. Culture Fair Intelligence Test-Scale 2 (Cattell).
3. General Mental Ability Test (Jalota).
4. Reasoning Ability Test (Dubey).
5. Space Relations (SAT) (Bennett, et al).
6. Adjustment Inventory (Asthana).
7. High School Personality Questionnaire - HSPQ (Cattell).
8. Academic Achievement in Five School Subjects (From School Records).

Statistical Treatment of Data

The descriptive statistics such as mean, median, mode, standard error, standard deviation, skewness, kurtosis were computed through computer for each variable included in the study which showed that the measures of the variables were normally distributed with minor variations. The relationships between the measures of the dimensions of adolescent thought and the measures of the independent variables, namely intelligence, reasoning ability, space relations, academic achievement, adjustment and personality traits were worked out by computing product moment correlations. One-way analysis of variance technique was used to determine the age and sex differences regarding the performance on Piaget Type Tasks during the formal-operational period at different age levels.

To identify the factorial structure of adolescent thought, the data on the 34 measures were put into 34 x 34 correlation matrix and subjected to factor analysis by Principal Axes Method. The computations were carried out through "MAYAD-1022 Computer"

at Computronics India, New Delhi, using P.A.-1 factor analysis programme from the "Statistical Package for the Social Sciences (SPSS)" by Nie, et al (1970).

Findings and Conclusions

Findings of the study and the conclusions drawn are presented here briefly:

- 1) The values of F-ratios computed between the five age groups with respect to their performance on ten Piaget Type Tasks were all found to be significant at .01 level (vide Table 10) and a gradual increase in mean performance at successive age levels was noticed (Table 11). Hence, it has been concluded that performance on Piaget Type Tasks increases with age during the formal-operational period.
- ii) The values of thirtyone t-ratios, out of the sixty t-ratios computed between the performances of boys and girls on ten Piaget Type Tasks at each of the five age levels as well as for the combined groups, were found to be significant. The boys showed superiority in all the cases of significant t-ratios (vide Tables 12 to 21) which led to the conclusion that boys perform either equal to or better than girls on Piaget Type Tasks at respective age levels.
- iii) The coefficients of correlation between the measures of intelligence both verbal and non-verbal and the variables of the

dimensions of adolescent thought were all found to be significant at .01 level (vide Table 23). It has been concluded, therefore, that measures of intelligence correlate significantly with the measures of the dimensions of adolescent thought.

iv) Looking through the analytical picture of the coefficients of correlations, it has been found that all the five measures of academic achievement were significantly correlated with the variables of the dimensions of adolescent thought except the dimensions of ratio and proportion and space visualization (vide Table 23). A conclusion has been arrived at that academic achievement has a significant bearing on the development of adolescent thought.

v) The coefficients of correlation between the measures of reasoning ability and space relations on one hand and the measures of the dimensions of adolescent thought on the other hand were all found to be significant at .01 level (vide Table 23). Thus, the measures of reasoning ability and space relations (measures of abstract thinking) prove to be the determinants of the development of adolescent thought.

vi) The values of all the coefficients of correlation between the measure of adjustment and the variables of the ten dimensions of adolescent thought were significant at .01 level (vide Table 23). It has been confirmed, therefore, that the development of formal thinking leads to better adjustment of the individual and vice versa.

Factor, Group Factor of Adolescent Thought and Social Factor. Keeping in view the significant factor loadings of the different factors, it has been concluded that the measures of intelligence, academic achievement, reasoning ability, space relations, adjustment and other personality traits cluster in specific constellations with the measures of the dimensions of adolescent thought explaining thereby the common variance operating among them.

Educational Implications

All over the world, education system is under fire from all sections of the society mainly because it is not delivering the goods properly, the emphasis has always remained on memorization. Thus, the qualitative development of the pupils remain hampered even upto adulthood. That is why, as discussed in Chapter III, majority of the students are found operating at concrete-operational level even at college stage. Hence, it is very essential to develop a curriculum and teaching methods at school level keeping in view, psychologically speaking, the structure of the adolescent thought determined empirically as in the case of the present study. The other implications of the findings of the study may be described as follows:

- 1) The development of the various traits of personality, such as obedience, feeling of security, emotional stability, self-discipline, relaxation, phlegm, abstract thinking, conscientiousness, outgoing-tendencies and adventurousness

should be given more emphasis as they help in the development of adolescent thought as found in this study.

iii) More attention should be paid towards the adjustment of the students at school level as it has been found to be, invariably, a correlate of the different dimensions of adolescent thought.

iii) Boys and girls should not be segregated in the classroom on the basis of sex as there does not exist a definite evidence regarding the superiority of the boys over the girls. Moreover, girls should be given more freedom and recognition, so that they may avail equal opportunities required for the development of adolescent thought.

iv) The age-related development of adolescent thought during adolescence does suggest that there should not be a large variation in age of the students studying in a particular class, otherwise, the classroom instruction can not be turned to a uniform frequency.

v) Since the measures of abstract thinking such as reasoning ability and space relations are found to be significantly related with the development of adolescent thought, exercises of abstract thinking should be included in day to day teaching which will promote the development of formal thought and consequently the cognitive functioning as a whole.

- vi) The results of factor analysis show the existence of a general intellectual factor of adolescent thought. The emphasis should, therefore, be given on the development of the overall abilities of the students, rather than on a special type of orientation only.
- vii) Lastly, an intellectual atmosphere should be created in the class rooms. It may also help in the development of the adolescent thought as the measures of intelligence, both verbal and non-verbal, are found to be significantly correlated with all the dimensions of adolescent thought irrespective of their form and content.

Problems for Further Research

The research on thinking processes and consequently on its structure is a very complex and imaginative phenomenon as the processes involved in thinking and the factors identified in the case of structure are not directly observable entities. Thus, the conclusions drawn remain purely hypothetical. Even then, they give way to an understanding of the functioning and of the vectors of mind with regard to thinking and structure of thought respectively. No study is complete in itself. It raises further queries regarding the issues involved in its investigation. The present study also raises some issues which are proposed to be undertaken in the near future.

- 1) The results of this study could not confirm the effect of sex on the development of adolescent thought, therefore, it

should be verified whether the structure of thought remains the same both in the case of boys and girls taken separately during adolescence.

- ii) The dimensions of adolescent thought are not found uniformly related with the various personality factors. Hence, the relationship between the personality factors and the dimensions of adolescent thought should be investigated more comprehensively controlling the other variables.
- iii) The development of adolescent thought has been found closely related with the increase in age levels. Thus, it has become a matter of deep concern to verify the invariance of the structure at different age levels, i.e., 11⁺, 13⁺, 15⁺, 14⁺ and 15⁺ years with the development of adolescent thought. It will also clarify whether the different abilities integrate or differentiate or develop independently with the increase in age.
- iv) The hump effect, witnessed on only few dimensions, should be further explored taking into account more tests related with the dimensions of adolescent thought for either accepting or rejecting the phenomenon.
- v) The possibility of the existence of the dimensions of adolescent thought other than those investigated in this study should be explored and their relationship with the dimensions already investigated should be studied.
- vi) The mathematical structure of the different school subjects should also be determined so that it could be matched with the cognitive structure of the pupils while framing the school curriculum.

1361907

Bibliography

- Abramowitz, G. (1975). Adolescent understanding of Proportionality. Ph.D. Thesis, Stanford University.
- Alexander, H.P. (1935). Intelligence, Concrete and Abstract. *British Journal of Psychological Monographs Supplements*, 12, 177.
- Asthana, L.T. (1968). *Manual of Direction & Norms for Adjustment Inventory*. Rupa Psychological Centre, Varanasi.
- Asthana, L.T. (1976). *Adjustment Inventory*. Rupa Psychological Centre, Varanasi.
- Aune, B. (1967). Thinking. In Paul Edwards (Ed.). The Encyclopedia of Philosophy. The Macmillan Company and the Free Press, New York. 8, 100.
- Bart, J.M. (1971). The Factor Structure of Formal Operations. *British Journal of Educational Psychology*, 41, 70-77.
- Bartlett, P.C. (1958). Thinking : An Experimental Study. George Allen and Unwin Ltd., London.
- Beard, R.M. (1957). An Investigation of Concept Formation Among Infant School Children. Ph.D. Thesis, Institute of Education, London.
- Beard, R.M. (1969). An Outline of Piaget's Developmental Psychology. Routledge and Kegan Paul Ltd., London.
- Bennett, et al. (1959). *Space Relations Test (SAT)*. Manassayan, New Delhi.

- Bennett, et al. (1939). Manual for the Differential Aptitude Tests (DAT). The Psychological Corporation, New York.
- Blasi, A. & Beffell, J.C. (1974). Adolescence and Formal Operations. *Human Development*, 17, 344-363.
- Bolton, W. (1972). The Psychology of Thinking. Methuen and Co. Ltd., London.
- Brainerd, L.L. (1973). Piaget's Theory of Intelligence. Prentice-Hall Inc., Englewood Cliffs, New Jersey.
- Brown, J. & Stephenson, L. (1933). A Test of the Theory of Two Factors. *British Journal of Psychology*, 23, 352-370.
- Brown, J. & Thomson, G.L. (1941). The Principles of Mental Measurement. Cambridge University Press, Cambridge.
- Bruner, J.W. et al. (1956). A Study of Thinking. John Wiley, New York.
- Burt, C. (1941). The Factors of the Mind : an Introduction to Factor Analysis in Psychology. Mac Millan, New York.
- Burt, C. (1944). The Structure of the Mind : A Review of the Results of Factor Analysis. *British Journal of Educational Psychology*, 13, 176-199.
- Cuswell, G.F. (1936). Law and Order of Thinking in Solving Problems. University of Chicago Press, Chicago.
- Case, R.B. & Collinson, J.H. (1962). The Development of Formal Thinking in Verbal Comprehension. *British Journal of Psychology*, 52, 103-111.

- Cattell, R.M. (1952). Factor analysis. Harper & Bros., New York.
- Cattell, R.M. & Nelloff, J.L. (1967). High School Personality Questionnaire HPQ. (Hindi Version by S.P. Kapoor & K.T. Mehrotra). The Psycho-Centre, New Delhi.
- Cattell, R.M. & Cattell, D.L. (1976). Handbook for the Jr. - High School Personality Questionnaire, (1976). The Psycho-Centre, New Delhi.
- Cattell, R.M. & Cattell, A.K. (1960). Handbook for the Jr. High School Personality Questionnaire, (1960). The Psycho-Centre, New Delhi.
- Cattell, R.M. & Cattell, A.K. (1964). Adult Pair Intelligence Test-Scale 2. The Psycho-Centre, New Delhi.
- Chiappetta, M.L. & Alfred, T.C. (1973). The Effectiveness of Verbal Label Training in Aiding Second Grade Pupils to Transfer their Classificatory Skill. Journal of Research in Science Teaching, 12, 2, 123-121.
- Clayton, V. & Overton, J.P. (1976). Concrete and Formal Thought Processes in Young Adulthood and Old Age. International Journal of Aging and Human Development, 7, 3, 237-245.
- Cloutier, R. & Goldschmid, M.L. (1976). Individual Differences in the Development of Formal Reasoning. Child Development, 47, 1097-1102.
- Cohen, J., Ed. (1964). Conflicts in Psychology. George Allen and Unwin Ltd., London.
- Dale, L.G. (1970). The Growth of Systematic Thinking: Replication and Analysis of Piaget's First Chemical Experiment. Australian Journal of Psychology, 22, 277-286.

- Delencos, H.H. (1969). The Development of Conservation in Aboriginal Children. *International Journal of Psychology*, 4, 255-269.
- Dewey, J. (1910). How We Think. D.C. Heath & Co., London.
- Dhalwal, A.G. (1977). Personality Correlates of Academic Over-Under Achievement. Guru Nanak Dev University, Amritsar, Punjab.
- Dubey, L.N. (1974). Manual of Reasoning Ability Test. National Psychological Corporation, Agra.
- Dubey, L.N. (1978). Reasoning Ability Test. National Psychological Corporation, Agra.
- Dudek, T.T. et al. (1969). Relationship of Piaget Measures to Standard Intelligence and Motor Scales. *Perception and Motor Skills*, 29, 351-362.
- Dulit, G. (1973). Adolescent Thinking a' la Piaget : The Formal Stage. *Journal of Youth and Adolescence*, 7, 281-301.
- Dunlop, D.L. & Fatso, F. (1975). A study of Abstract Preferences in Problem Solving Tasks and Their Relationship to Abstract Ability and Formal Thought. Paper Presented at the Annual Meeting of the National Association for Research in Science Teaching, Los Angeles, California.
- Elkind, D. (1962). Quantity Conceptions in College Students. *Journal of Social Psychology*, 57, 459-465.
- Elkind, D. (1977). Children's Discovery of Conservation. In Gruber & Vonèche (Eds.). *The Development of Conservation*. Routledge Kegan Paul Ltd. London.

- Alkousay, A. A. (1935). The Visual Perception of Space.
British Journal of Psychological Monographs
Supplements, 20, 89.
- Encyclopaedia Britannica. (1956). Encyclopaedia Britannica,
Inc. Chicago. 22, 133.
- Erikson, E. E. (1958). Identity : Youth and Crisis.
Faber & Faber, 3 Queen Square, London.
- Eysenck, H. J. (1939). Critical Notice of 'Primary Mental
abilities' by L. L. Thurstone. British Journal
of Educational Psychology, 9, 270-276.
- Eysenck, H. J. (1967). Intelligence Assessment : A Theoretical
and Experimental Approach. British Journal of
Educational Psychology, 37, 81-98.
- Eysenck, H. J. (1979). The Structure and Measurement of
Intelligence. Springer-Verlag, Berlin
Heidelberg, New York.
- Flavell, J. H. (1963). The Developmental Psychology of Piaget.
Kinget, J. Van Nostrand Company Inc., Toronto,
Canada.
- Freud, S. (1935). Psychoanalysis for Teachers and Parents.
Amerson, New York.
- Freud, S. (1949). An Outline of Psychoanalysis, Norton, New York.
- Fruchter, B. (1967). Introduction to Factor Analysis.
Affiliated East - West Press Pvt. Ltd., New Delhi.
- Garrett, L. B. (1971). Statistics in Psychology and Education.
Vohla, 12200 and 12200 Pvt. Ltd., Bombay,
212-246.

- Germain, J.C. et al. (1976). The Personality of the Child and the Utilization of Operative Thought. *Infance*, 4-5, 399-405.
- Graybill, L.L. (1974). A Study of Sex Differences in the Transition from Concrete to Formal Thinking Patterns. *Dissertation Abstract International*, 34, 7, 3988 A.
- Graybill, L.L. (1975). Sex Differences in Problem-Solving Ability. *Journal of Research in Science Teaching*, 12, 4, 341-346.
- Green, D.L., Ed. (1971). Measurement and Piaget. McGraw-Hill Book Co., New York.
- Gruber, E. & Vonèche, J.F. Ed. (1977). The Essential Piaget. Routledge & Kegan Paul Ltd., London.
- Guerin, H.O. (1973). A Quasi - Simplex and Alpha Factor Analysis of Piaget Based Logical Operations. *Dissertation Abstract International*, 33, 10, 6512.
- Guilford, J.P. (1936). Psychometric Methods. McGraw-Hill Book Co., New York.
- Guilford, J.P. (1940). Human Abilities. *Psychological Review*, 47, 367-399.
- Guilford, J.P. (1956). The Structure of Intellect. *Psychological Bulletin*, 53, 267-293.
- Gunnels, F. (1967). A Study of the Development in Logical Judgements in Science of Successful and Unsuccessful Problem Solvers in Grades Four Through Nine.
- Harman, L.L. (1960). Modern Factor Analysis. The University of Chicago Press, Chicago.

- Hatab, A. (1964). The Definition and Measurement by Verbal Methods of the Ability to Think Critically. M.A. Thesis. Institute of Education, London.
- Mathway, J.S. (1975). The Unique Contribution of Piagetian Measurement to Diagnosis, Prognosis and Research of Children's Mental Development. In John and Della Hodgill (ed.). Piagetian Research: Compilation and Commentary. NFER Publishing Company, London, 1976. 3, 174.
- Haggings, P.A. & Gaito, A.J. (1971). Maturity of Formal Operational Thought in Adolescents. Proceedings of 79th Annual Convention of the American Psychological Association, Washington, D.C.
- Kolczinger, K.J. (1933). Statistical Methods for Talents in Education. Ginn & Co., Boston.
- Kolczinger, K.J. & Terman, L.L. (1933). Comparison of Two Factorial Analyses. *Psychometrika*, 3, 45-60.
- Nowe, A. (1974). Formal Operational Thought and the High School Science Curriculum. Paper Presented at the National Association for Research in Science Teaching, Annual Meeting, Chicago.
- Humphrey, G. (1951). Thinking. Methuen & Co., Ltd. London.
- Inhelder, B. & Piaget, J. (1958). The Growth of Logical Thinking from Childhood to Adolescence. Routledge & Kegan Paul Ltd., London.
- Jackson, S. (1968). The Growth of Logical Thinking in Normal and Sub-Normal Children. *British Journal of Educational Psychology*, 38, 255-258.

- Jalota, T. (1973). Manual of Directions for the General Mental Ability Test. The Psycho-Centre, New Delhi.
- Jalota, T. (1976). General Mental Ability Test. The Psycho-Centre, New Delhi.
- Joshi, J.N. (1970). The Development of Algebraic Concepts during Secondary School Years. Ph.D. Thesis, Punjab University, Chandigarh.
- Joyce, D.E. (1977). A Study of Formal Reasoning in Elementary Education Majors. *Science Education*, 61, 2, 153-158.
- Juraschak, J. (1975). The Performance of Prospective Teachers on Certain Piagetian Tasks. Dissertation Abstract International, 35, 9, 3909A.
- Karplus, H. & Arons, A.B. (1976). Implication of Accumulating Data on Levels of Intellectual Development. *American Journal of Physics*, 44, 4, 396.
- Karplus, H. et al. (1973). Intellectual Development Beyond Elementary School I: Logic: The Testimony of Cognitive Piaget. Center for Studies in Science, University of California, Berkeley, California.
- Karplus, H. & Karplus, A.B. (1970). Intellectual Development Beyond elementary School I: Deductive Logic. *School Science and Mathematics*, 70, 398-405.
- Keating, D.F. (1975). Precocious Cognitive Development at the Level of Formal Operations. *Child Development*, 46, 276-280.
- Keating, D.F. & Caramazza, A. (1975). Effects of Age and Ability on Syllogistic Reasoning in Early Adolescence. *Developmental Psychology*, 11, 6, 837-842.

- Kozka, J. (1955). *Developmental Psychology: A Study of the Growth of Intelligence*. New York: Holt, Rinehart & Winston.
- Kelley, T.L. (1928). *Intelligence in the Mind of Man*. Stanford University Press, Stanford, California.
- Thun, H. (1976). Relation of Two Piagetian Stage Tests to IQ. *Developmental Psychology*, 12, 2, 102-105.
- Koffka, E. (1933). Principles of Gestalt Psychology. Harcourt, Brace & World, New York.
- Kohlberg, L. & Gilligan, T. (1971). The Adolescent as a Philosopher: The Discovery of the Self in a Post-Conventional World. *Unadulus*, 100, 4, 1051-1055.
- Kohler, W. (1945). *Gestalt Psychology*. Liver-right, New York.
- Lawson, J.E. (1975). Sex Differences in Concrete and Formal Reasoning Ability as Measured by Manipulative Tasks and Written Tasks. *Science Education*, 59, 3, 397-405.
- Lawson, J.E. (1976). Formal Operations and Field Independence in a Heterogeneous Sample. *Perceptual and Motor Skills*, 42, 981-982.
- Lawson, J.E. (1977). Relationships Among Performances on Three Formal Operations Tasks. *The Journal of Psychology*, 95, 233-241.
- Lawson, J.E. & Blake, M.J. (1976). Concrete and Formal Thinking Abilities in High School Biology Students as Measured by Three Separate Instruments. *Journal of Research in Science Teaching*, 13, 3, 227-235.
- Lawson, J.E. & Renner, J.W. (1974). A Quantitative Analysis of Responses to Piagetian Tasks and its Implications for Curriculum. *Science Education*, 58, 4, 545-559.

- Tawson, T. & Rimmer, J. S. (1975). Relationship of Science Subject Matter and Developmental Levels of Learners. *Journal of Research in Science Teaching*, 12, 4, 347-358.
- Lee, L. S. (1971). The concomitant development of cognitive and moral modes of thought : a test of selected predictions of Piaget's Theory. *Genetic Psychology Monographs*, 83, 93-146.
- Langol, L. A. & Shuell, J. R. (1972). Exclusion of Irrelevant Factors : The Pendulum Problem. *Science Education*, 56, 66-70.
- Libre, J. J. & Levine, J. T. (1976). Adolescent Reasoning : The Development of the Ability to Control Variables, "Advancing Education Through Science Education Programmes", Report No. 9 Lawrence Hall of Science, Berkeley.
- Longoot, P. (1963). Analyse Statistique De Trois Tests Genetiques. *Bulletin du L'Institut National*, 1963, 4, 310-337.
- Lovell, K. (1961). A Follow-up Study of Inhelder and Piaget's : The Growth of Logical Thinking. *British Journal of Psychology*, 52, 143-153.
- Lovell, K. (1972). Developmental Processes in Thought. In *Intelligence in Young Children and Its Development* (Selected Readings) Edited by Jean Piaget, The Macmillan Co., New York, 100-116.
- Lovell, K. & Butterworth, J. B. (1966). Abilities Underlying the Understanding of Proportionality. *Mathematics Teaching*, 37, 5-9.
- Lovell, K. & Ogilvie, J. (1977). Conservation of Substance : Growth of Conservation Volume. In The Essential Piaget, *ibid.*
- MacArthur, G. S. (1968). Some Differential Abilities of Northern Canadian Native Youth. *International Journal of Psychology*, 3, 43-51.

- Martel, J.J. (1974). An Analysis of Piaget's Logical-Mathematical Model for the Period of Formal Operations. *Dissertation Abstracts International*, 34, 6, 3152-3153.
- Martoreano, S.C. (1977). A Developmental Analysis of Performance on Piaget's Formal Operations Tasks. *Developmental Psychology*, 13, 6, 666-672.
- McKinnon, J. & Renner, J. (1971). Are Colleges Concerned with Intellectual Development. *American Journal of Physics*, 39, 1047-1052.
- Mealings, H.J. (1961). Some Aspects of Problem Solving in Science. M.A. Thesis, Institute of Education, University of Birmingham.
- Meeks, G. & Meeks, V. (1971). The Development of Formal Thought as Shown by Application of the Oscillations of a Pendulum: A Study of Adolescence. *Journal of Research in Science Teaching*, 8, 219-223.
- Mills, L.C. & Dean, P.M. (1969). Problem Solving Methods in Science Teaching. Teacher Education, Bureau of Publications, Teacher's College, Columbia University, New York.
- Hodgill, T. and Celia (1974). Piagetian Research Compilation and Commentary Volume No. 3. NFAH Publication Co. Ltd., Windsor Berks.
- Nie, N.H. et al. (1970). Statistical Package for the Social Sciences. McGraw-Hill Book Company, New York, 110-210.
- Nordland, et al. (1974). A Study of Levels of Concrete and Formal Reasoning Ability in Disadvantaged Junior and Senior High School Science Students. *Science Education*, 50, 4, 869-875.
- Osicki, K.J. (1973). Affective and Cognitive Development: Comparison of Need Achievement and Risk Level with Piagetian Levels of Cognitive Development for Two Socio-Economic Groups. *Dissertation Abstracts International*, 34, 6, 3152-3153.

- Paterson, D.G. et al. (1930). Minnesota Mechanical Ability Tests. Minnesota University Press, Minneapolis.
- Peel, J.A. (1960). The Pupil's Thinking. Old Bourn, London.
- Peel, J.A. (1966). Psychology and the Teaching of Science. British Journal of Educational Psychology, Nov.66.
- Piaget, J. (1935). Judgement and Reasoning in the Child. Harcourt, Brace & World, New York.
- Piaget, J. (1950). The Psychology of Intelligence. International Universities Press, New York.
- Piaget, J. (1951). Play, Dreams and Imitation in Childhood. Norton, New York.
- Piaget, J. (1952). The Origins of Intelligence in Children. International Universities Press, New York.
- Piaget, J. (1954). The Construction of Reality in the Child. Basic Books, New York.
- Piaget, J. (1962). The Language and Thought of the Child. Kegan Paul, London.
- Piaget, J. (1967). Biology and Knowledge. University Chicago Press, Chicago.
- Piaget, J. (1970a). Genetic Epistemology. Columbia University Press, New York.
- Piaget, J. (1970b). Science of Education and the Psychology of the Child. Orion, New York.
- Piaget, J. (1972). Intellectual Evolution from Adolescence to Adulthood. Human Development, 15, 1-12.

- Piaget, J. & Inhelder, B. (1977). Conservation of Substance, Weight and Volume. In The Essential Piaget, ibid.
- Rajput, M.D. (1975). A Study of the Scheme of Proportion Among Certain Groups of Adolescent Pupils. Unpublished M.Ed. Thesis, Ujjain University, Bhopal.
- Raven, J.C. (1973). The Development of A Test of Piaget's Logical Operations. Science Education, 57, 3, 377-385.
- Stinner, J. L. & Stafford, D.G. (1972). Teaching Science in the Secondary School. Harper & Row, New York.
- Ross, H.J. (1973). Some Empirical Parameters of Formal Thinking. Journal of Adolescence, 169-177.
- Boyce, J. L. (1960). A Synthesis of Experimental Designs in Program Research. Journal of General Psychology, 63, 295-303.
- Santhan, V. S. (1978). An Analogy Between Piagetian Grouping of Thought and Group Theory in Algebra. Indian Educational Review, 13, 3, 81-85.
- Seyre, A. & Daniel, J.D. (1975). Piagetian Cognitive Development and Achievement in Science. Journal of Research in Science Teaching, 12, 2, 165-174.
- Schwarz, P.A. & Krug, M. (1972). Ability Testing in Developing Countries - A Handbook of Principles and Techniques. Praeger Publishers, London.
- Schwebel, M. (1975). Formal Operations in College Freshmen. Journal of Psychology, 91, 133-141.
- Shayer, M. (1979). Has Piaget's Construct of Formal Operational Thinking Any Utility? British Journal of Educational Psychology, 49, 255-276.

- Shaver, M. & Hylam, L. (1978). The Distribution of Piagetian Stages of Thinking in British Middle and Secondary School Children. II - 14 to 16 Year-Olds and Sex Differentials. *British Journal of Educational Psychology*, 48, 62-70.
- Comerville, J.C. (1974). The Pendulum Problem : Patterns of Performance Defining Developmental Stages. *British Journal of Educational Psychology*, 44, 3, 256-231.
- Spearman, C. (1927). The Abilitation of Man. Macmillan Co., New York.
- Staver, J.R. and Sabal, D.L. (1979). The Development and Construct Validation of a Group - Administered Test of Formal Thought. *Journal of Research in Science Teaching*, 16, 6, 535-544.
- Stephenson, W. (1931). Tetrad - Differences for Non-Verbal Subtests. Tetrad-Differences for Verbal Subtests. Tetrad-Differences for verbal Sub-tests Relative to Non-Verbal Subtests. *Journal of Educational Psychology*, 22, 167-185, 255-257, 334-350.
- Stephenson, W. (1955). The Study of Behavior. University of Chicago Press, Chicago.
- Stephens, W.B. et al. (1969). The Development of Reasoning, Moral Judgement and Moral Conduct in Retardates and Normals. Department of Health Education and Welfare, Temple University Philadelphia, Washington D.C.
- Thomson, G.H. (1951). The Factorial Analysis of Human Ability. Houghton Mifflin Co., New York.
- Thurstone, L.L. (1935). The Vectors of Mind. University of Chicago Press, Chicago.

- Thurstone, L.L. (1938). Primary Mental Abilities. Psychometric Monograph, 1, 121.
- Tisher, R.P. (1971). The Development of Some Science Concepts : A Replication of Piaget's Studies with Pupils in New South Wales Country High School. Unpublished B.A. (Hon.) Thesis, University of New England, Armidale, New Wales.
- Titchener, W.B. (1898). Postulates of a Structural Psychology. Philosophical Review, 7, 440-466.
- Tuddenham, R.O. (1970). A Piagetian Test of Cognitive Development. In J. Cockrill (ed), On Intelligence. Methuen & Co. Ltd, London.
- Upadhyaya, G.P. (1973). A Study of Intellectual Development and its Relationship with Intelligence and Achievement of 10th Grade Science pupils. Unpublished M.A. Dissertation, University of Rajasthan, Jaipur.
- Vaidya, N. (1964). A Study of Problem Solving in Science Among Certain Groups of Adolescent Children. M.A. Thesis Institute of Education, London.
- Vaidya, N. (1968). Problem Solving in Science. G. Chand & Co. New Delhi.
- Vaidya, N. (1971). The Impact Science Teaching. Oxford, & IBJ Publishing Co., New Delhi.
- Vaidya, N. (1974). How Children Discover Knowledge. Oxford, IBJ Publishing Co., New Delhi.
- Vaidya, N. (1975). A Study of some Aspects of Thinking Among Science Students of Adolescent Age. Ph.D. Thesis, University of Rajasthan, Jaipur.

- Vaidya, N. (1979). The Growth of Logical Thinking in Science During Adolescence. Oxford & IB Publishing Co., New Delhi.
- Vaidya, N. & Misra, H.M. (1975). The Role of Hypotheses in Solving Problems of Science. *The Rajasthan Board Journal of Education*, 11, 4, 1-10.
- Vaidya, N. & Gandhi, T.C. (1978). Hyp effect as Observed during Problem Solving. *Educational Trends*, 13, 2, 39-62.
- Valentine, J.L. (1975). Performance on two Reasoning Tests in Relation to Intelligence, Divergence and Interference Awareness. *British Journal of Educational Psychology*, 45, 198-205.
- Vernon, P.E. (1961). The Structure of Human Abilities. Methuen & Co., Ltd. London.
- Vinacke, J.L. (1968). The Psychology of Thinking. Prentice Hall Inc. N.J. & N.Y. New York.
- Waite, J.B. (1978). A Study Comparing College Science Students' Performance on Piagetian Type Tasks, Including Cross-Cultural Comparisons. *Dissertation Abstracts International*, 35, 9, 8954 A.
- Walker, N.A. et al. (1979). Written Piagetian Task Instruments: Its Development and Use. *Science Education*, 63, 3, 211-230.
- Watson, J.B. (1913). Psychology as the Behaviourist Views It. *Psychological Review*, 20, 2, 158-177.
- Watson, J.B. (1934). Behaviourism. Norton, New York.
- Weeks, R.T. (1973). The Relationship of Grade, Sex, Socio-economic Status, Scholastic Aptitude and School Achievement to Formal Operations Attainment in a Group of Junior High School Students. *Diss. Abst. International*, 34, 5-A, 2405.

Wertheimer, M. (1923). Laws of Organization in Perceptual Forms (Translated and condensed) In J.D.Allis, A Source Book of Gestalt Psychology. Harcourt, Brace & World, New York, 1939.

Wertheimer, M. (1945). Productive Thinking. Harper & Row, New York.

Wosny, C.D. (1974). The Effects of Culture and Education on the Acquisition of Formal Operational Thinking. Dissertation Abstracts International, 34, 7, 4016.

Wosny, C.D. & Fox, D.L. (1978). The Effects of Task Differences on the Assessment of Formal Operational Thinking. Paper Presented at the Annual Meeting of the American Educational Research Association, Washington, D.C.

Yudin, L. (1966). Formal Thought in Adolescence as a Function of Intelligence, Child Development, 37, 697-708.

Yudin, L. (1967). The Nature of Adolescent Thought. Adolescence, 2, 137-151.

Winnipeg

APPENDIX (i)

Test of Piaget Type Tests

Part I

Time : 45 Min.

Name of the Student

.....

Boy or Girl

.....

Father's Name

.....

Name of the School

.....

Class

.....

Section

.....

Roll No.

.....

Date

.....

TASK NO. 1

A gardener gave a bunch of flowers of different types to his son and asked him to classify the flowers into different groups. The boy put the roses in one group, cherrill in the other and so on. Then the gardener asked him to further sub-classify the roses into different categories. The boy put the red roses in one category, the white roses in the other and so on. Now you go through the task given ahead and try to perform as demanded.

Here are given some geometrical figures on the next page (Fig. 1). Each figure has been assigned a number for its identification. Classify those figures into three main categories in such a way that similar figures come under one category. You are to write only the numbers of the figures under the categories I, II and III to which they belong:

Category I

Category II

Category III

Now, further sub-classify each category into three sub-categories in such a way that similar figures of each category come under the three sub-categories of the same:

Category I
Sub-categories
A B C

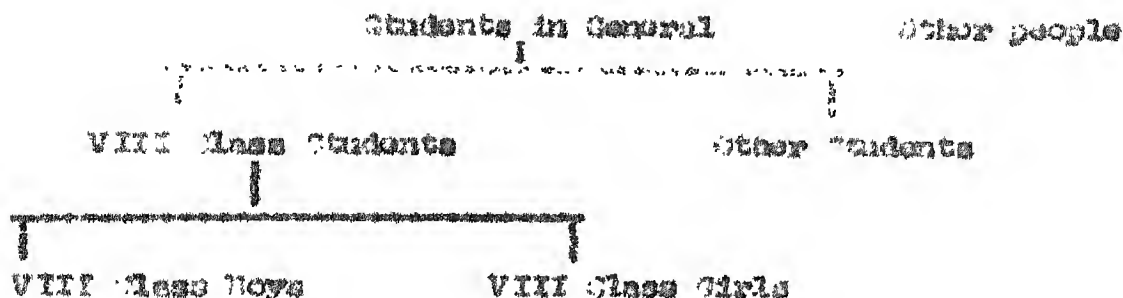
Category II
Sub-categories
A B C

Category III
Sub-categories
A B C

TABLE 10. 2

Look through the Classification Hierarchy given below (Fig. 2) and with the understanding of it answer the questions given ahead.

Fig. 2



1. If VIII class boys and VIII class girls are grouped together, what name will you give to the group formed? _____
2. If VIII class boys are grouped with VIII class students and then both in VIII class are grouped with students in general, what name will you give to the group formed? _____
3. If VIII class students are grouped with students in general and then both in combination are grouped with VIII class boys, what name will you give to the group formed? _____
4. If VIII class boys are grouped with a class without students, what name will you give to the group formed? _____
5. If VIII class boys are grouped with VIII class girls and then VIII class boys are taken out from the group, what name will you give to the group left behind? _____
6. If VIII class boys are grouped with VIII class boys, what name will you give to the group formed? _____
7. If VIII class boys are grouped with VIII class students, what name will you give to the group formed? _____

TABLE No. 3

Three sets of numbers and algebraic symbols having two columns each are given below (Fig. 3). In each set you will find some relationship between the entities of the two columns. Try to understand the relationship and fill in the blanks given in each set :

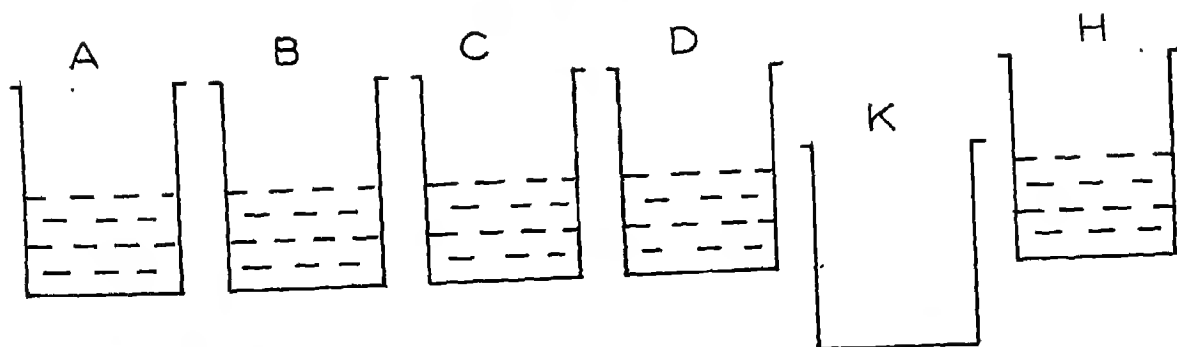
Fig. 3

1	2	3
2 \rightarrow 8	$-7 \rightarrow -12$	$5 \times 2 = 10$
6 \rightarrow 9	$-13 \rightarrow -18$	$5 \times 3 = 15$
10 \rightarrow 13	$-19 \rightarrow -24$	$5 \times () = 20$
14 \rightarrow ()	$-25 \rightarrow ()$	$() \times 5 = 25$
() \rightarrow ()	$() \rightarrow ()$	$() \times () = ()$
$A \rightarrow A+3$	$-A \rightarrow -A-3$	$5 \times A = 5A$
$A+4 \rightarrow A+7$	$-A-6 \rightarrow -A-11$	$5 \times (A+1) = 5A + 5$
$A+B \rightarrow ()$	$-A-B \rightarrow ()$	$5 \times (A+B) = ()$
() \rightarrow ()	() \rightarrow ()	$() \times () = ()$
() \rightarrow B	() \rightarrow B	$5 \times () = B$
() \rightarrow ()	() \rightarrow ()	$() \times () = ()$

TASK (P). 4

Four beakers 1, 2, 3 and 4 are placed on a table (Fig. 4). The fifth beaker 5 is placed a bit away. All the beakers are filled up with different chemical reagents. There is one empty beaker K. You performed an experiment with these chemicals one day. You took out the chemical reagent from some of the 1, 2, 3 and 4 beakers and put into the beaker 5. Then he took out the reagent from beaker 1 and put it also into the beaker 5. Thus, the contents of the beaker 5 became yellow in colour. What experiments will you perform to find out the reagents which on putting together have made the yellow colour appear? Describe all possible experiments.

Fig. 4

Examples:Experiment No. 1

Took out the chemical reagent from the beaker A and put it into the empty beaker K. Then took out the chemical reagent from the beaker 1 and put it also into beaker K.

Experiment No. 2

Took out the chemical reagents from the beakers B and C and put them into the empty beaker K. Then took out the chemical reagent from the beaker 4 and put it also into the beaker K.

Similarly, you write down the other possible experiments.

Experiment No.

Experiment No.

Experiment No.

Experiment No.

Experiment No.

Experiment No.

Experiment No.

Experiment No.

Experiment No.

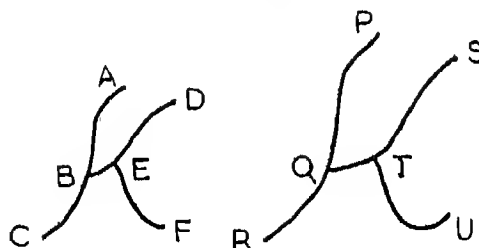
Experiment No.

Experiment No.

Ratio No. 8

A letter "T" is given (Fig. 3) in two different sizes - one small and the other large. It is given that the lengths of the arms of the small letter and the large letter are in the ratio of 2:3. On the basis of this information answer the questions given below:

Fig. 3



1. If length of the arm AB is 4 cms., what will be the length of the arm PQ?
2. If length of the arm QR is 8 cms., what will be the length of the arm ST?
3. If length of the arm AS is 16 cms., what will be the length of the arm PT?
4. If length of the arm RC is 10 cms., what will be the length of the arm UT?
5. If length of the arm BE is 9 cms., what will be the length of the arm TU?

.....

.....

.....

.....

.....

REPLACEMENT (11)

Test of Piaget Type Tasks

Part II

Time : 45 min.

Name of the Student

Boy or Girl

Father's Name

Name of the School

Class

Section

Roll No.

Date

TEST NO. 6

The children of your age are very curious to know about the things in their environment. Number of questions come to their minds whose answers they do not know. For example, Mohan, a child of your age asked the following questions about the Sun:

1. Is ^{the} Sun a ball of fire in reality ?
2. Why the Sun does not fall on the Earth ?
3. Can we live without ^{the} Sun ?
4. What is the temperature of the Sun ?

Thus many questions might have been coming to your mind also. You please write down as many questions as you can whose answers you do not know, about (a) Bicycle, and (b) Cow.

(a) Bicycle

(b) Cow

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

TASK NO. 7

There is given a board having traced nine squares upon it (Fig. 6). The top three squares (A,B,C) were painted blue, the middle three squares (D,E,F) were painted white and the bottom three squares (G,H,I) were painted red. When they were painted second time it so happened that the left three squares (A,D,G) got painted red, the middle three squares (B,E,H) got painted blue and the right three squares (C,F,I) got painted white. Thus, the different colours got mixed and changed in the various squares as follows:

1. The square which was painted blue and red or vice-versa looked gray in colour.
2. The square which was painted blue and white or vice-versa looked light blue in colour.
3. The square which was painted red and white or vice-versa looked pink in colour.

You please write down the name of the colour in each square of which it looked like after the two paints.

Fig. 6

A	B	C
D	E	F
G	H	I

PART III. B

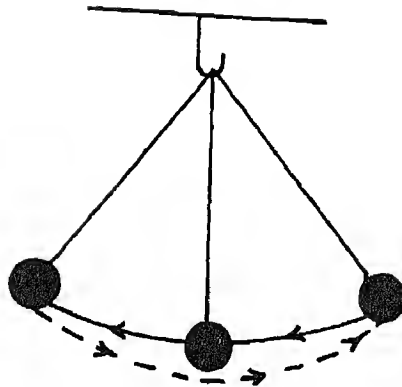
Then asked Mohan what are the factors upon which depends the drying up of a wet handkerchief? Mohan told him that this question could have many answers such as :

1. Nature of the stuff, i.e., cotton, silk, etc.
2. Length
3. Thickness
4. Colour
5. Temperature
6. Season, etc.

Then Ram asked Mohan to prove the effect of these factors with the help of the experiments. Mohan suggested as follows:

Suppose I am to prove that the drying up of a wet handkerchief depends upon the length of it. I shall take three handkerchiefs of the same stuff, same thickness, same colour, etc., but with different lengths only. I shall make them equally wet and put them in the sun or shade. The time taken by each handkerchief to dry up will be noted with the help of a watch. If the handkerchief having the smallest length dries up first and the one having the largest length dries up at the last then it is proved that the drying up of a wet handkerchief depends upon the length of it, otherwise not. Similarly, the effect of the other factors can be proved through the experiments. Now you please solve the problem given ahead:

Fig. 7



A simple pendulum is shown in the Fig. 7 above. The bob of the pendulum oscillates on both sides of the centre. The movement of the bob from the centre to ^{the} left end, back to the centre, then to the right end and back to the centre is called the one oscillation of the pendulum. You please write down the factors upon which depends the time taken in one oscillation of the pendulum and prove the effect of each factor through experiments.

- Factors :
1. _____
 2. _____
 3. _____
 4. _____
 5. _____

Experiment No.

1. The first experiment was conducted in the laboratory of the Department of Physics, University of California, San Diego, in the summer of 1964. The purpose of this experiment was to determine the effect of the concentration of the solution on the rate of reaction.

Experiment No.

2. The second experiment was conducted in the laboratory of the Department of Chemistry, University of California, San Diego, in the summer of 1964. The purpose of this experiment was to determine the effect of the concentration of the solution on the rate of reaction.

Experiment No.

3. The third experiment was conducted in the laboratory of the Department of Physics, University of California, San Diego, in the summer of 1964. The purpose of this experiment was to determine the effect of the concentration of the solution on the rate of reaction.

Experiment No.

4. The fourth experiment was conducted in the laboratory of the Department of Chemistry, University of California, San Diego, in the summer of 1964. The purpose of this experiment was to determine the effect of the concentration of the solution on the rate of reaction.

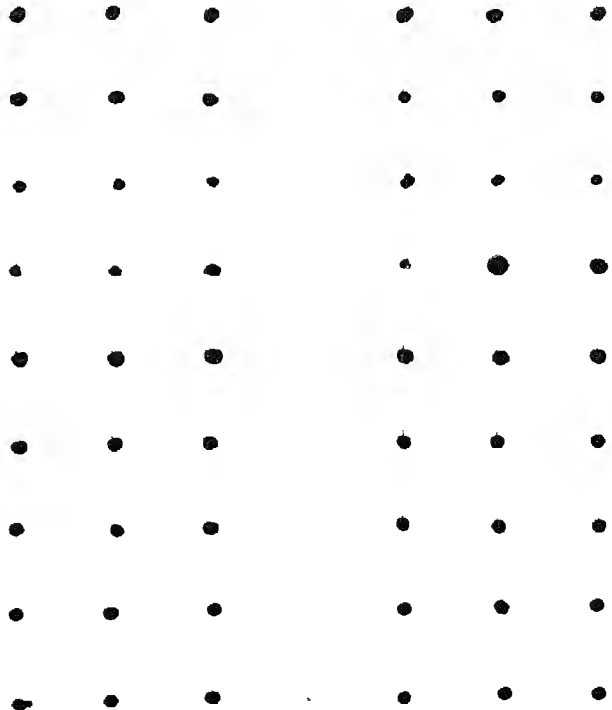
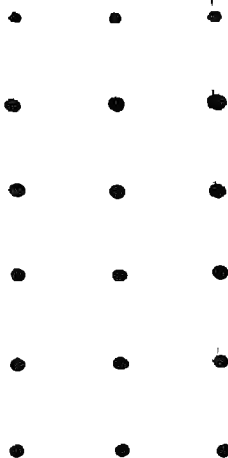
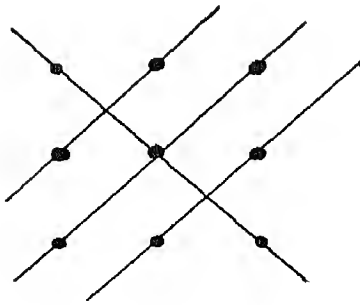
Experiment No.

5. The fifth experiment was conducted in the laboratory of the Department of Physics, University of California, San Diego, in the summer of 1964. The purpose of this experiment was to determine the effect of the concentration of the solution on the rate of reaction.

TABLE NO. 9

A number of sets of nine dots are given below (Fig. 8). You please try to draw four straight lines in such a way that all the nine dots are touched by one or the other line. Repeat this exercise with as many different ways as you can but the number of lines should not exceed four and no dot should remain untouched.

Fig. 8



EXERCISE NO. 10

Read, understand and then answer the questions given below:

1. There is a 10 metre long rod of wood out of which 1 metre rod is cut after every minute. How much time will it take to be cut into pieces of 1 metre length each?

2. Some ducks are swimming in a pond in a straight line. Two ducks are on the front side, two in the middle and two on the backside. How many ducks are there in all?

3. Ram has four friends. Three of his friends are having names as Bryan, Mohan and Chit. What is the name of the fourth friend?

4. There is a blind man. He can see upto 100 metres through one eye. How far will he be able to see through both the eyes?

5. A donkey has two horns. How many horns will be having eight donkeys?

APPENDIX (III)

REPRINTED FROM INDIAN EDUCATIONAL REVIEW JULY 1978 1980/1

An Analogy between Piagetian Grouping of Thought and Group Theory in Algebra

TEG SINGH SANDHU

*Junior Research Fellow
Examination Research Unit
NCERT, New Delhi*

It was in 1920, that J. Piaget, while working with Dr. Theophile Simon in Paris, to develop a standardized test, found that the children's incorrect answers were fascinating. He found that the same wrong answers occurred frequently in children of the same age. From the analysis of the nature of the mistakes, he came to the conclusion that older children were not only just brighter than younger ones, but also the thought of younger children was qualitatively different from that of older ones. On the basis of the quality of the responses at different age level, Piaget divided intellectual development into four major periods : (i) Sensorimotor period (0 to 2 years) ; (ii) Pre-operational period (2 to 7 years) ; (iii) Concrete operational period (7 to 11) years ; and (iv) Formal operational period (11 years and above). The characteristics of these four stages are given below

1. Sensorimotor Period (0 to 2 Years)

During this period language appears and symbolic functioning makes its acquisition possible.

2. Pre-Operational Period (2 to 7 Years)

This was further sub-divided into two periods : (a) Ranging from (2 to 5 years), and (b) Ranging from (5 to 7 years).

At the stage (a) the child fails to construct hierarchical arrangements because after a short while he forgets the defining property which he has used to form a collection. At the stage (b) the child can construct a hierarchy because he can use a defining property to determine which

objects go in a collection. But he cannot understand inclusion relations

3. Concrete Operational Period (7 to 11 Years)

At this stage, concrete operations are organized. Operational grouping of thought concerning objects can be manipulated or known through senses. Child can correctly answer questions concerning inclusion, because of his ability to think of original classes and their derivatives at the same time. But the child fails to comprehend the same relations when imaginary classes are involved.

4. Formal Operational Period (11 Years and above)

At this stage, actions are internalized. The child can operate on operations. He can compensate mentally for transformations in reality. Mental operations have reached a high degree of equilibrium thus effecting a second degree grouping of operations.

From the above mentioned characteristics it becomes clear that 'grouping of thought' starts at the third stage of development of intellect. Our further discussion, primarily, will be focussed on the third stage, i. e. concrete operational period.

Let us now see both the terms 'operation' and 'grouping' from the point of view of logicians as well as psychologists, to compute the similarities between the two viewpoints. According to psychologists operation means to arrive at a real functioning of intelligence or to revert the thinking in terms of actions. While according to logicians or mathematicians 'operation' means a symbol, representing an action which could be realized.

Analysis of a mathematical nature has since long recognized the interdependence of operations, constituting certain well defined systems, such as groups. The notion of a 'group' is applied to series of whole numbers, to spatial or temporal structures, to algebraic operations, etc. Psychologically, a 'grouping' consists of a certain form of equilibrium of operations, i. e. actions which are internalized and organized in complex structures and the individual is to describe this equilibrium.

Group Theory and Piagetian Grouping of Thought

The important question here is to determine the conditions of the

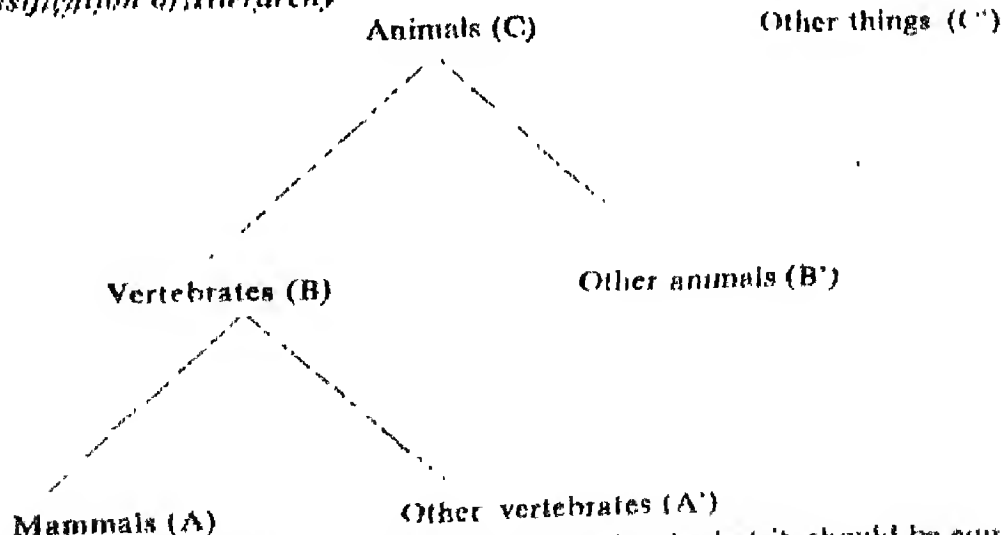
equilibrium (i.e. mobile equilibrium, since operations are actions, the equilibrium of operational thought is in no way a state of rest) in order to be able subsequently to examine how it is formed genetically. These conditions are four in number in the case of a group of mathematical order and five in the case of grouping of a qualitative order. We can make a comparative study of the two types of orders as follows :

In mathematics, a set G equipped with a composition, denoted multiplicatively is called a group, if

- (i) Composition is associative, i.e.
 $a(bc) = (ab)c \forall a, b \text{ and } c \in G.$
- (ii) There exists an identity for the composition, i.e.
 $a \in G$, there exists an element $e, \in G$ such that $a \cdot e = a = ea$
- (iii) Every element is invertible, i.e. for every element $a \in G$ there exists an element $b \in G$, such that $ab = e = ba.$

Taking each conditions separately we can compare it with the operations of 'grouping of thought'. For this purpose let us take an example of the sort of classification hierarchy of animals, which the children of the age of 7-11 years, or above can construct (according to Piaget's experiments).

Classification of Hierarchy



I. The first condition of a group in algebra is that it should be equipped with a composition, i. e. any two elements of a group may be combined and thus produce an element of the same group. This property is usually referred as closure.

I'. According to Piaget, this first condition expresses the possibility of

coordinating operations in 'grouping of thought', i. e. two distinct classes may be combined into one comprehensive class which embraces them both. For example, if we combine the mammals with other vertebrates we get the general class of vertebrates. This may be written as $A + A' = B$ or if we combine mammals with vertebrates we get all the vertebrates. We may write this as $A + B = B$. This property describes aspects of the child's ability to understand a hierarchy. If he can mentally construct a large class by combining its sub-classes.

II. Second condition of the group is that composition is associative, i. e.

$$a(bc) = (ab)c \quad a, b, c \in G$$

II'. The property of associativity in 'grouping of thought' may well be illustrated from the given example of classification hierarchy.

Suppose we want to combine three classes such as mammals, vertebrates and animals, i. e. A, B and C. We cannot add all the three at a time, since the operator (combining) is binary, i. e. it can be applied only to two elements at a time. Therefore, there are at least two ways of adding A, B and C. *First*, We can add mammals to vertebrates and get vertebrates and then adding vertebrates to the animals we get animals in general, i. e.

$$i) (A + B) + C = C$$

Second, We can do by combining the mammals with the group of vertebrates and animals and finish with the same results, i.e. animals in general,

$$ii) A + (B + C) = C$$

From first and second, we get

$$(A + B) + C = A + (B + C)$$

Thus associativity expresses the fact that the child can combine classes in different orders and can realize that the results are equivalent.

III. The third condition of the group in algebra is that there exists an identity for the composition, i. e. for every element a belonging to G , there exists an element e , $e \in G$ such that

$$ae = a = ea$$

III'. In the child's classification of 'grouping of thought' identity states that there is a special element in the system (the 'nothing' element) which when combined with any of the other elements produces no change. From our example, cited before, if we combine to 'nothing' element with mammals, the result will be mammals. More correctly, if we do not combine the mammals with any of the other classes, then of course we still have the mammals.

IV. The fourth condition of the group is that every element is invertible, i. e.

$$ab = e \Rightarrow ba \quad \forall a, b, e \in G$$

IV'. The reversibility in 'grouping of thought' can be illustrated from the given example. Suppose we combine the mammals with all other vertebrates, we get all vertebrates. But if we take away inverse or negation of all the other vertebrates except mammals then again we are left with mammals. This type of operation expresses the aspect of class inclusion. Such a reasoning underlies the child's ability to say that there are more vertebrates than mammals, i. e. mammals are more included in the class of vertebrates.

V. The fifth property of 'grouping of thought' is unique. It has several aspects. One of them has to do with special identity elements. Suppose we combine the class of mammals with itself, the result is mammals. We may write this as :

$$A + A = A$$

In this equation A functions as an identity element. Piaget calls this a tautology. Another aspect is resorption. If we combine the class of mammals with the class of vertebrates the result is vertebrates. We may write this :

$$A + B = B$$

Here too, A functions as an identity element. In a sense, this is another way of looking at inclusion relations.

These are some of the aspects of grouping, described by the processes underlying the child's classifications.

REFERENCES

- GINSBURG HERBERT and SYLVIA OPPER, *Piaget's Theory of Intellectual Development*, Prentice-Hall, Inc., New Jersey, 1969
- PIAGET, J., *Judgment and Reasoning in the Child*, Trans. M. Warden, New York, Harcourt, Brace & World, Inc., 1926
- , *The Psychology of Intelligence*, Routledge & Kegan Paul Ltd., London
- , *The Origins of Intelligence in Children*, Trans. M. Cook, International University Press, New York, 1952
- SHANTI NARAYAN, *A Textbook of Modern Abstract Algebra*, S. Chand & Co., 1967

APPENDIX (iv)

Table of Variables

Cr. No.	Code	Variable
1.	IV	Intelligence verbal
2.	IV	Intelligence non-verbal
3.	ADJ	Adjustment
4.	CA	Reserved/Outgoing
5.	CT	Concrete thinking/Abstract thinking
6.	CS	Emotionally less-stable/Emotionally stable
7.	CP	Phlegmatic/Excitable
8.	CA	Obedient/Assertive
9.	CP	Cautious/Reckless
10.	CA	Dependent/Conscientious
11.	CP	Shy/Adventurous
12.	CT	Tough-minded/Tender-minded
13.	CT	Jealous/Circumspect
14.	CT	Secure/Insecure
15.	CT ₂	Group-dependent/Self-sufficient
16.	CT ₃	Uncontrolled/Self-disciplined
17.	CT ₄	Relaxed/Tense
18.	CT ₁	Space relations
19.	CT ₁₁	Reasoning ability
20.	CT ₁₂	Classification
21.	CT ₁₃	Grouping of thought
22.	CT ₁₄	Generalization to arithmetical and algebraic symbols.
23.	CT ₁₅	Permutations and combinations
24.	CT ₁₆	Ratio and proportion
25.	CT ₁₇	Formulation of probing questions
26.	CT ₁₈	Interpretation and coordination of information
27.	CT ₁₉	Stating and testing hypotheses
28.	CV	Space visualisation
29.	CV	Grasping the essence of the problem
30.	AA ₁	Academic achievement in Maths.
31.	AA ₂	Academic achievement in Science.
32.	AA ₃	Academic achievement in English
33.	AA ₄	Academic achievement in Punjabi
34.	AA ₅	Academic achievement in Hindi

APPENDIX (V)

Educational Trends

Volume 13, No. 2, July, 1978

Lump Effect as Observed During Problem Solving

T. S. Sandhu

Regional College of Education,
Ajmer

N. Valiya

Regional College of Education,
Ajmer

Background

By its very nature, the investigation of human thought right from infancy to adulthood is a complex venture. Its experimental investigation through the techniques of problem solving is of recent origin. Problem solving takes place as soon as the problem is perceived by the problem solver and is aimed at to reach the goal. The problem is supposed to be not only new and novel but also at the same time, there is supposed to be no direct solution available to the problem solver at the time of its presentation. Moreover, according to K. Duncker (1945), it is also assumed that the problem solver possesses the needed information for solving it. Problem solving, as a technique of investigation, has been utilized very widely in research studies pertaining to the diversified range of conceptual schemes of thought in the fields of general psychology and the newly emerging literature on science education. When search for clarity is made, overall confusion intervenes when problem solving is defined as 'combining the essentials of two isolated experiences' Maier (1930) or as 'the integrated activity of perception, memory, recall, association, generalization and reconstruction of ideas' Welch (1972). Even when seen in its specific context, problem solving ability is said to vary very widely, for example, the 'simple finding of exceptions' Hazlitt (1930) at one end, to the 'formal reasoning of a complex nature' Inhelder (1960) at the other end. This bleak situation still further worsens when problem solving situations used to evoke thinking vary very widely. Examples are dogs, cats and rats in puzzle boxes and mazes as well as human beings solving advanced problems involving fundamental concepts of mass, length, space and time.

The Past Literature

Past literature in this area can be classified in terms of studies in concept

formation, problem solving and when all lumped from the view point of teaching methodology, science education. Beard (1957), Hull et al (1961), Smoke (1961), Hausman and Kassanin (1961) and Bruner et al (1962) have investigated concept formation over the years but hardly any work seems to have been done on the evolution of concepts in relation to their functional and quantitative aspects. Heider (1928), Maier (1930), Durkin (1937), Mumford (1937), Duncker (1945) Husell (1956), Wheeler (1958) and Vaidya (1964) have investigated the phenomenon of problem solving with widely different problems having clear cut solutions. Here, too, there is no consensus regarding the age at which formal reasoning begins. In the area of science education, Denstasche (1943), Oakes (1947), Kyle (1950), Szekely (1950), Kruglak (1951), Whellock (1953), Butt (1957), Banks (1958), Mealings (1961), Neal (1961), Stendler (1951), Carpenter (1963), Horton (1963), Muthulingam (1963), Peel (1965) and Vaidya (1971) have investigated problem solving in science from different standpoints, making in the process the comparison of findings stemming from the various studies impossible. Consequently, there is no sharp theoretical framework available for seeing varied researches on problem solving. Yet there is one net gain of this long term effort that any researcher in this area is in a position to know the kind of road yet to be travelled with a view to cut and stitch concepts for maximum educational development with minimum of effort and cost.

The Main Study

It was undertaken to investigate certain aspects of thinking through the medium of problem solving among science students of adolescent age duly matched on intelligence and socio-economic status. One of its side aims was to study errors as they occur in solving a set of seventeen problems when presented individually in two sessions. These problems were mostly Piagetian in flavour and involved constant differences, summation, algebraic generalization, proportion, repeated structurings and restructurings, use of insight, proposing tests, combinational grouping, formulating problematic situations and stating as well as testing hypotheses.

Method of Procedure

Sample and subjects : A sample of 200 students, 100 boys and 100 girls ranging from age (10.5 to 11.5) to (14.5 to 15.5) in years corresponding to the grades VI through X, was selected and matched on the basis of intelligence test and socio-economic status (Jaloti and Kuppaswamy). Seventeen problems, each containing a continuous chain of reasoning, were administered individually in two sessions. These seventeen problems were further analysed in terms of thinking

processes, judged necessary to solve these problems which were later on reclassified into seventeen schemes of thought. Coefficients of reliability and validity for the entire problem solving test were determined which, according to Guilford (1956), were found to be within the range of acceptable limits.

Noticing the Hump Effect

It was observed that adolescent pupils have committed a large number of errors while engaged in the acts of problem solving. The dominant errors (shared by more than 20 percent of these pupils) were further found to increase with age before their frequency finally fell. It is of interest to highlight this finding because whereas understanding increased with age, the individual errors, contrary to expectations, too, appear to have suffered their ups and downs before finally declining with age. Why should it happen? Is it the case of an adolescent playing with figures thoughtlessly or arbitrarily in the hope of being favoured with good luck? Is it his care to respond to the varied test items in any manner he likes, regardless of consequences and meanings? Is it the case of lack of seriousness on his part? Is it his case of being caught between the horns of a dilemma and getting muck? Is it the case of hot chase trying hard to choose in haphazard directions as if in the manner of closing in on the problem? Does it illustrate that mastery of a thought process is through a path : uphill, thorny and often erratic? Or does the adolescent regress as if on an adventurous Piagetian journey during which he is trying hard to educate out himself, thinking that the right path to concept development lies in flourishing on experimental failures or a problem solving situation in which either understanding suffers a dip or errors a hump? Alternatively, is it a fact of rubbing his schemes of thought wrongly, especially when he has personal reservations about his self acquired knowledge in contrast to school learning which does not set right his firmly held self centred thoughts? Lastly is it the case that he chooses to be very romantic in his computations when confronted with a problem situation leading to a chaos? These are some of the stray notions which strike while having encountered this elusive phenomenon in several contexts :

- (i) When the answer to the test item is contained in the problem itself.
- (ii) When the test item needs an arithmetical or algebraic symbol.
- (iii) When the count is kept of total number of trials or errors on individual steps.
- (iv) When the same step is suggested again after having undertaken other steps in the furtherance of solution, namely, resting points.

- (v) Lastly, when the count is kept of total number of arbitrary as well as extraneous considerations brought into the problematic situation during problem solving.

Illustrations of the Phenomenon

Selected data involving 'Hump Effect' are now presented in respect of certain thought processes where dominant errors on them undertook unusual courses.

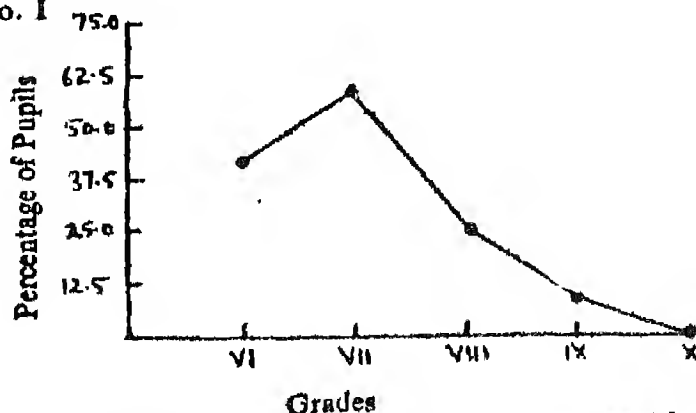
Table I

N=200

S. No.	Description of the Process	S. No of Problem & Process	Gradewise percentage of pupils committing errors				
			VI	VII	VIII	IX	X
1.	What is the height of Mohan ?	1(2)	40	57.5	25	10	0
2.	Generalization to Algebraic Symbol	2(13)	47.5	42.5	87.5	75	55
3.	What is the total distance when the man goes four times around the rectangle ?	6(37)	12.5	27.5	17.5	12.5	2.5
4.	Suppose a donkey has two horns. How many horns in all have eight donkeys ?	10(63)	27.5	20	42.5	52.5	37.5
5.	Beakers Problem	13	67.5	85	72.5	65	57.5
6.	What is the combined real depth of the fish when seen from above as well as from below ?	14(83)	12.5	25	20	42.5	5
7.	Proposing tests problem	16	35	47.5	62.5	27.5	2.5

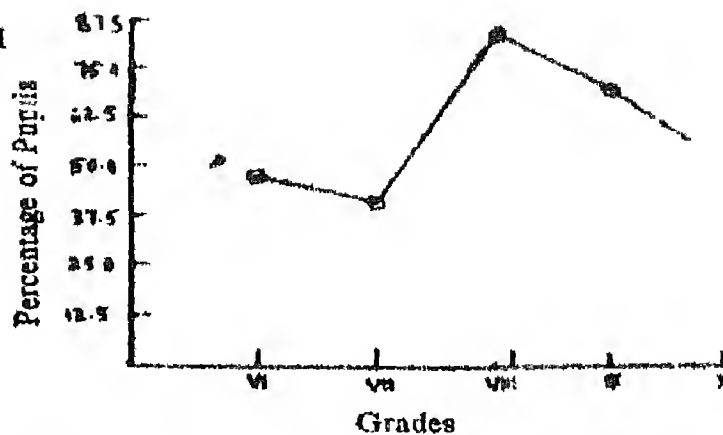
Consider now the graphical illustrations of the data presented above.

Fig. No. I



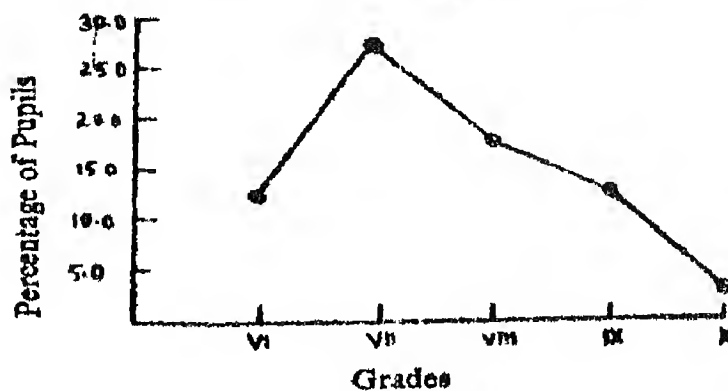
Hump of dominant error on process No. 2 of Problem No. 2

Fig. No. II



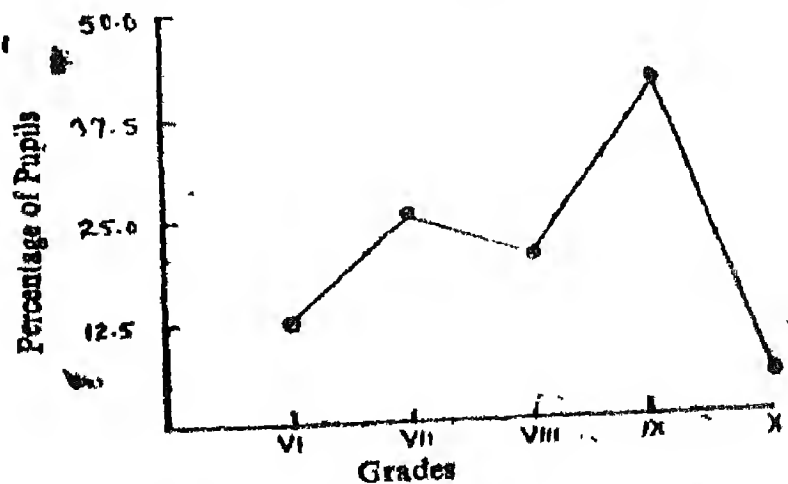
Hump of dominant error on Process No. 13 of Problem No. 2

Fig. No. III



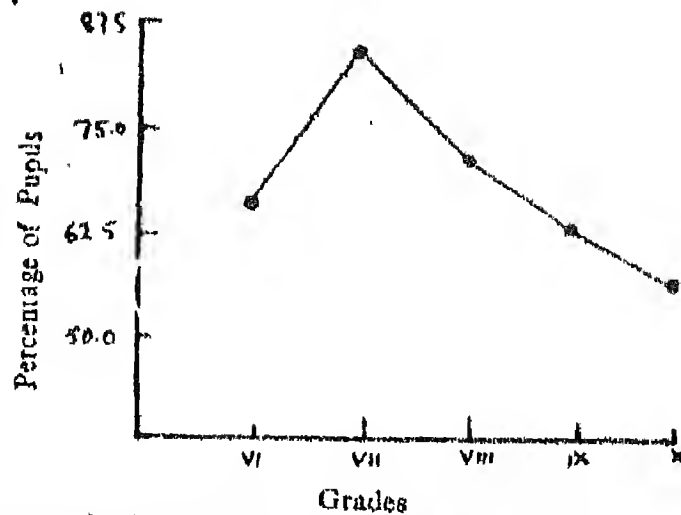
Hump of dominant error on Process No. 37 of Problem No. 6

Fig. No. VI



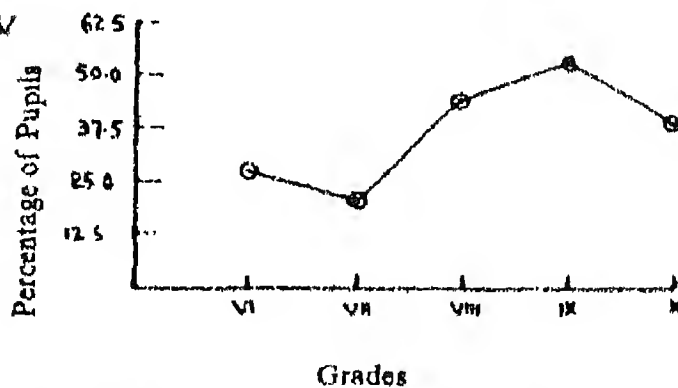
Hump of dominant error on Process No. 63 of Problem No. 10

Fig. No. V



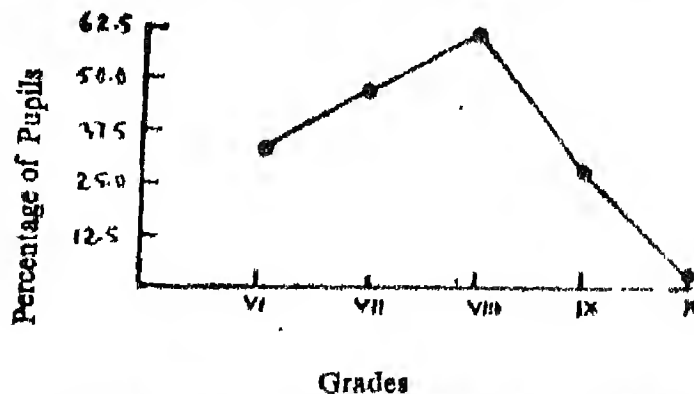
Hump on the incidence of arbitrary errors arising due to the failure to accept the demand of the problem No. 14

Fig No. IV



Bi-Hump of the dominant error on Process No. 83 of Problem No. 14

Fig. No. VII



Hump on the arbitrary errors arising due to the failure to grasp the essence of the Problem No. 16

Hump Effect During the Mastery of Thought Processes

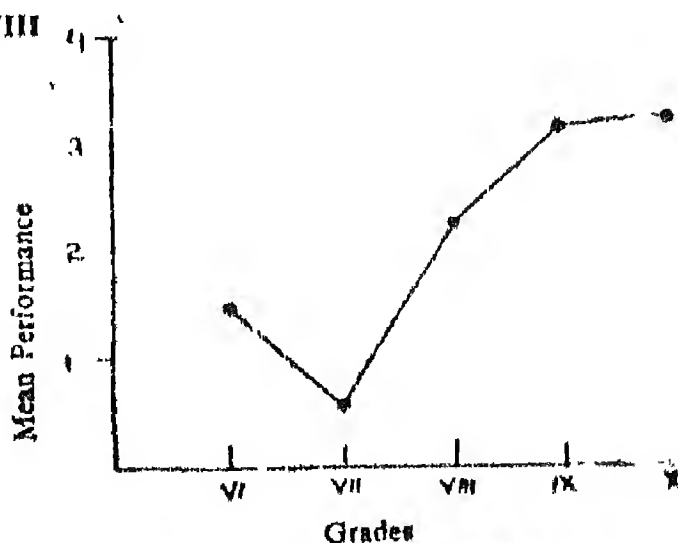
When data were further looked into, similar effect appeared, while the pupils were manifesting mastery on the various thought processes aggregated together as judged by their mean scores on respective problems. Data in the context are presented below :

Table II

S. No.	Description of the problem	S. No. of the Problem	Mean Scores at Different Grades				
			VI	VII	VIII	IX	X
1.	Height Problem	1	1.45	.53	2.25	3.13	3.20
2.	Digital Problem (Combinatorial)	9	2.67	2.45	5.35	5.40	7.78
3.	Formulating Questions Problem	12	5.67	9.13	7.40	9.65	11.48
4.	Fish Problem	14	.05	.28	2.56	2.30	4.90

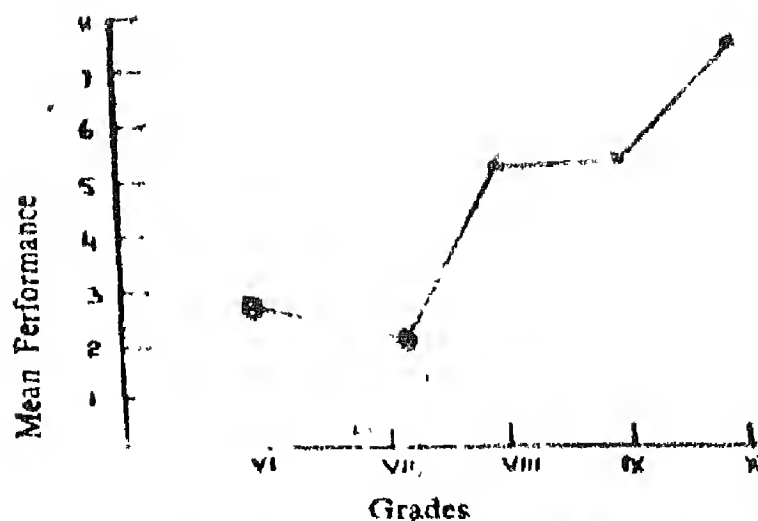
Now consider another set of graphical illustrations, showing hump effect in another context on mean scores of problems as referred to above.

Fig. No. VIII



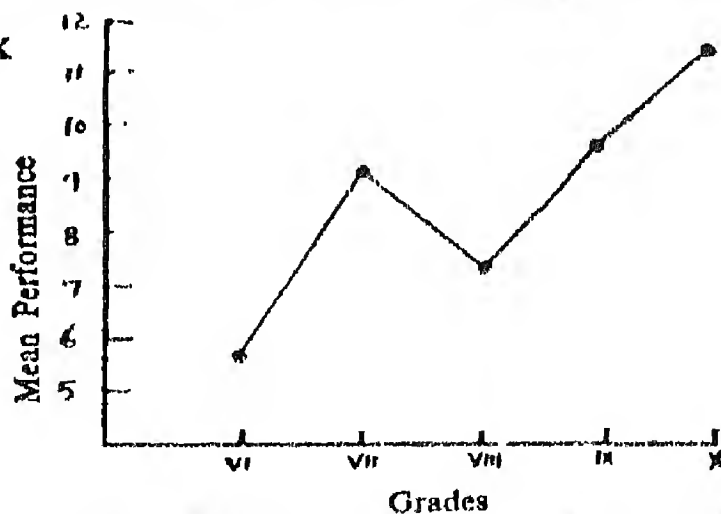
Hump in the mastery of thought processes on Problem No. 1

Fig. No. IX



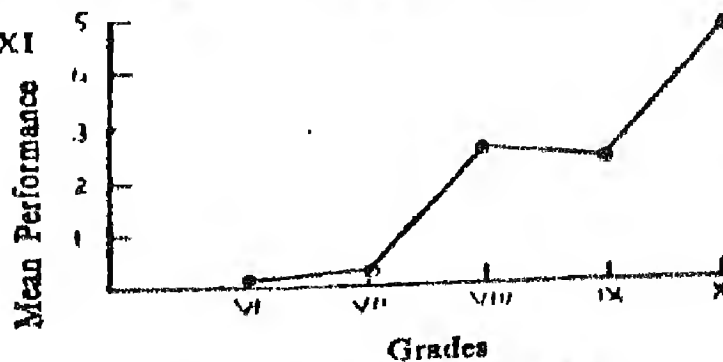
Hump in the mastery of thought processes on Problem No. 7.

Fig. No. X



Hump in the mastery of thought processes on Problem No. 12

Fig. No. XI



Hump in the mastery of thought processes on Problem No. 14

Sandhu (1978) while using Valdy's as well as other problems with class as the unit of sample in the actual classroom situations in Delhi encountered the same phenomenon among adolescent pupils studying in grades VI through X. The descriptions of various dimensions of adolescent thought along with the mean performances, on problems used in this study, at different grades are given below :

Table III

S. No.	Dimension of Adolescent Thought	Mean Performances at Different Grades				
		VI	VII	VIII	IX	X
1.	Hierarchical Classification	.60	14.60	24.20	19.66	32.00
2.	The Concept of Ratio	.13	.00	.53	.40	1.43
3.	Information Processing	2.20	3.40	5.40	4.33	7.68
4.	Formulation of Problematic Situations	9.33	11.46	10.06	16.20	16.31
5.	Hypotheses Testing	.06	.13	.60	.00	.87

Consider now the graphical illustrations reflecting the hypothesized phenomenon as shown by mean scores on the above dimensions of adolescent thought as well.

Fig XII

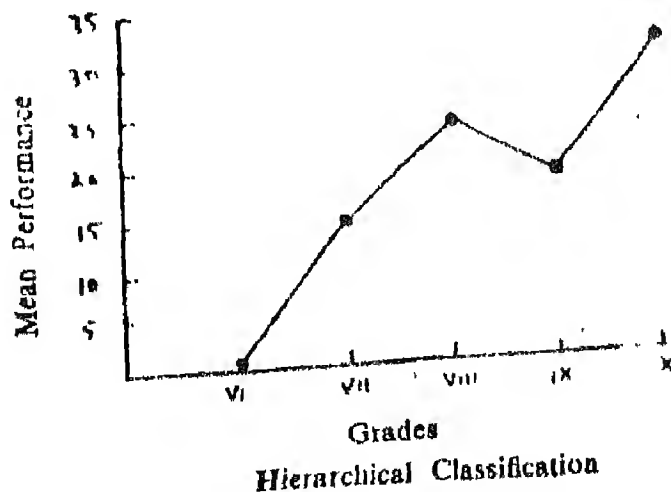


Fig. XIII

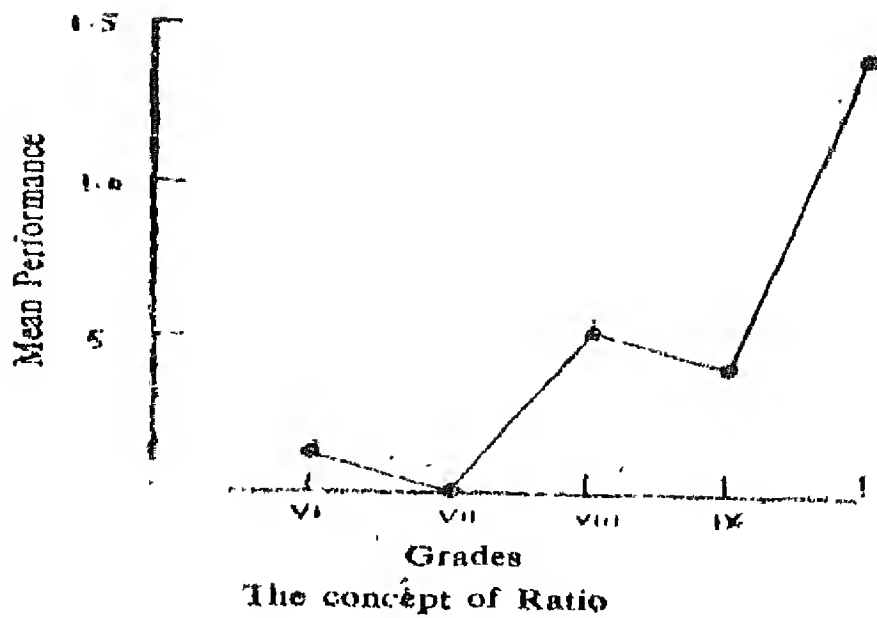
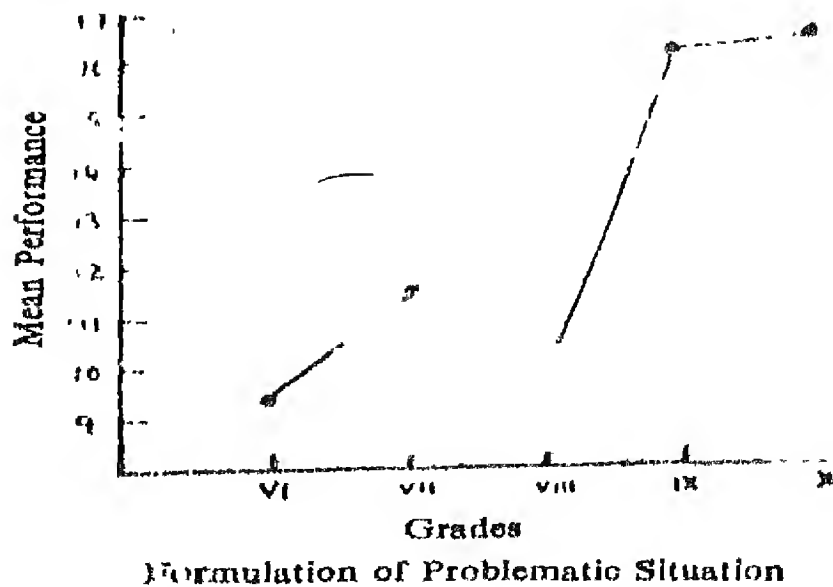


Fig. XIV



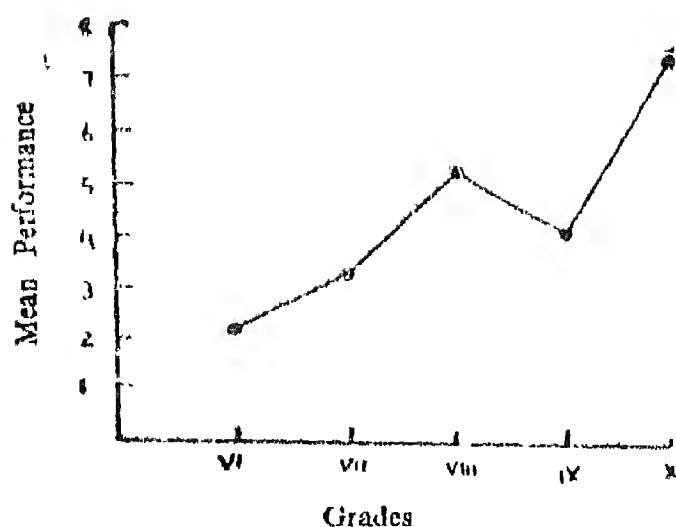
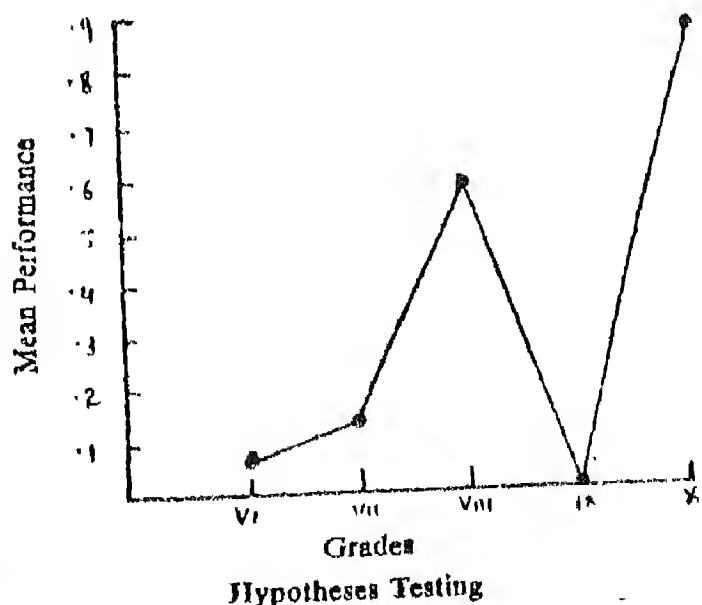


Fig. No. XVI

Information Processing



Hypotheses Testing

Additional Support from Literature

Due to the paucity of global literature in education and psychology in a provincial town like Ajmer, it is not possible to document this phenomenon if encountered earlier by other workers. It appears that Piaget and Inhelder, (1977), Lovell and Ogilvie, (1977) and David Elkind (1977) did encounter this phenomenon but missed

referring to it in their studies. Then data taken from *The Essential Piaget* when seen in the context of the hypothesized phenomenon indicated the same.

Table IV

Percentage success in Tests on the Conservation of Substance, Weight and Volume.

(According to Piaget and Inhelder)

Age	5	6	7	8	9	10	11
Substance (Transitional)	0	16	4	4	4	---	---
Weight (Transitional)	0	4	0	8	12	8	4
Volume (Transitional)	0	0	0	28	12	20	4

Table V

Percentage Success in Tests on the Conservation of Substance (N=322) and Weight (N=364)

(According to Lovell & Ogilvie)

Tests	Substance (Transitional)	Weight (Transitional)
Class I Age 7-8	23	5
Class II Age 8-9	12	36
Class III Age 9-10	15	20
Class IV Age 10-11	9	13

Table VI

Percentage success in tests on Conservation of Substance, Weight and Volume at different age levels (N = 25)

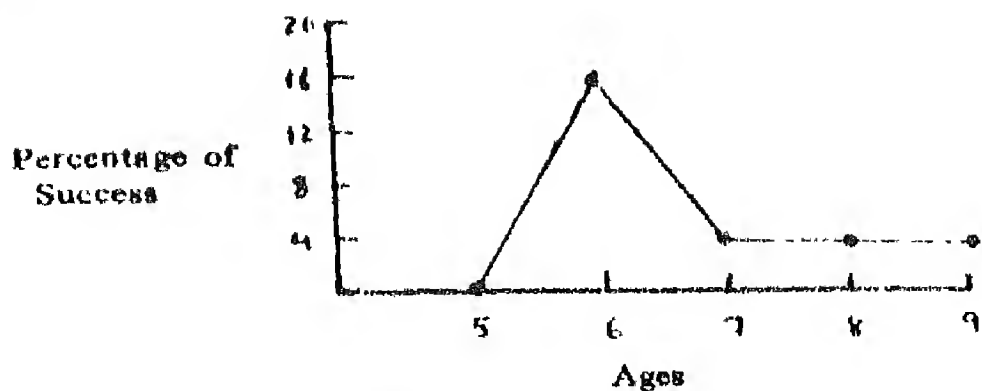
(According to David Elkind)

Age	5	6	7	8	9	10	11
Substance (Conservation)	19	51	70	72	86	94	92
Weight (Conservation)	21	52	51	44	73	89	78
Volume (Conservation)	0	4	0	4	4	19	25

Before considering the graphical illustrations based upon the above mentioned data, it is of interest to mention, that their subjects were, comparatively speaking, at the lower stages of mental development, namely, pre-operational and concrete operational as defined by Piaget. Secondly, special attention has been paid to the frequencies against the transitory stage of solution on concepts of conservation of substance, weight and volume.

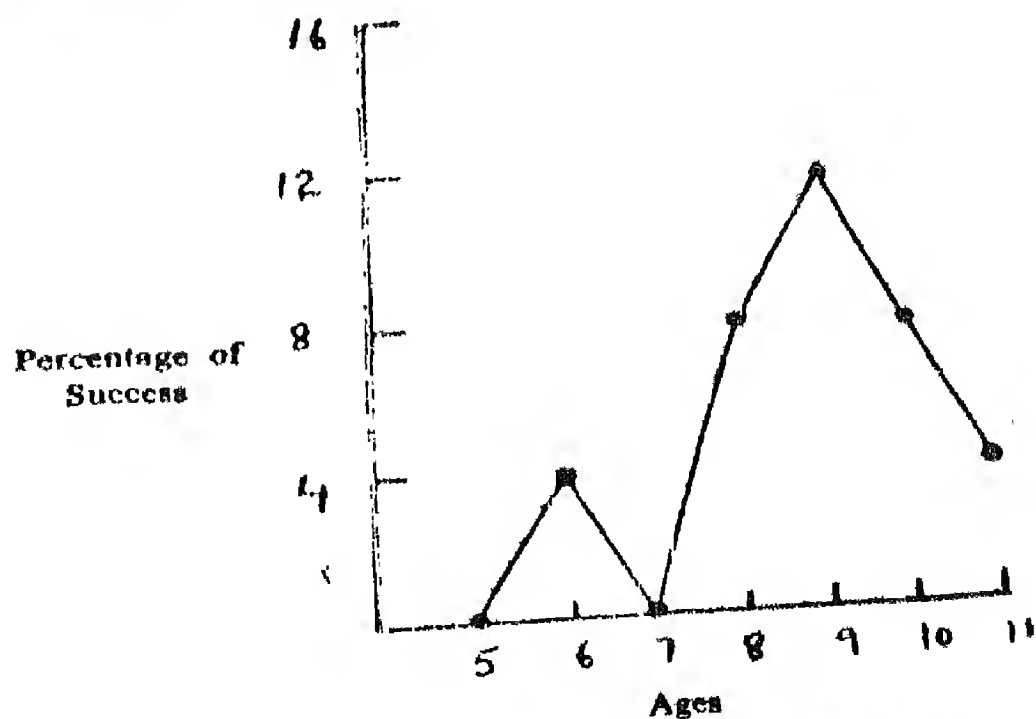
Now consider the graphical illustrations of the data described above.

Fig No. XVII



Conservation of Substance (Transitional)
According to Piaget and Inhelder

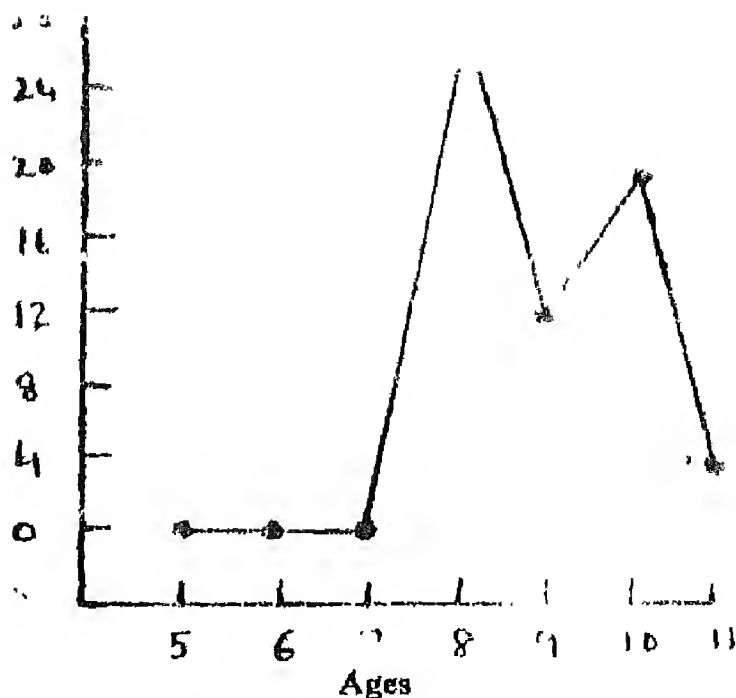
Fig. XVIII



Conservation of Weight (Transitional)
According to Piaget and Inhelder

Fig.

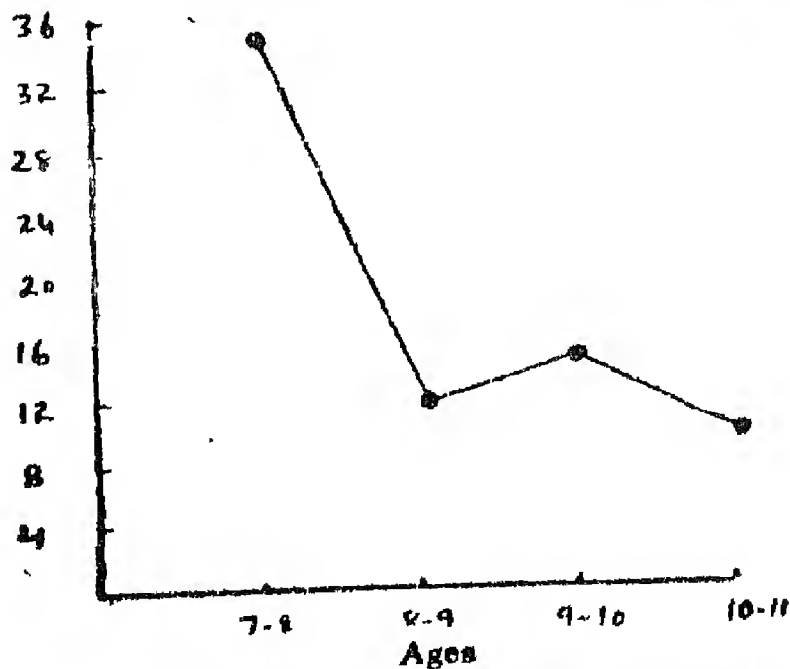
Percentag of
Success



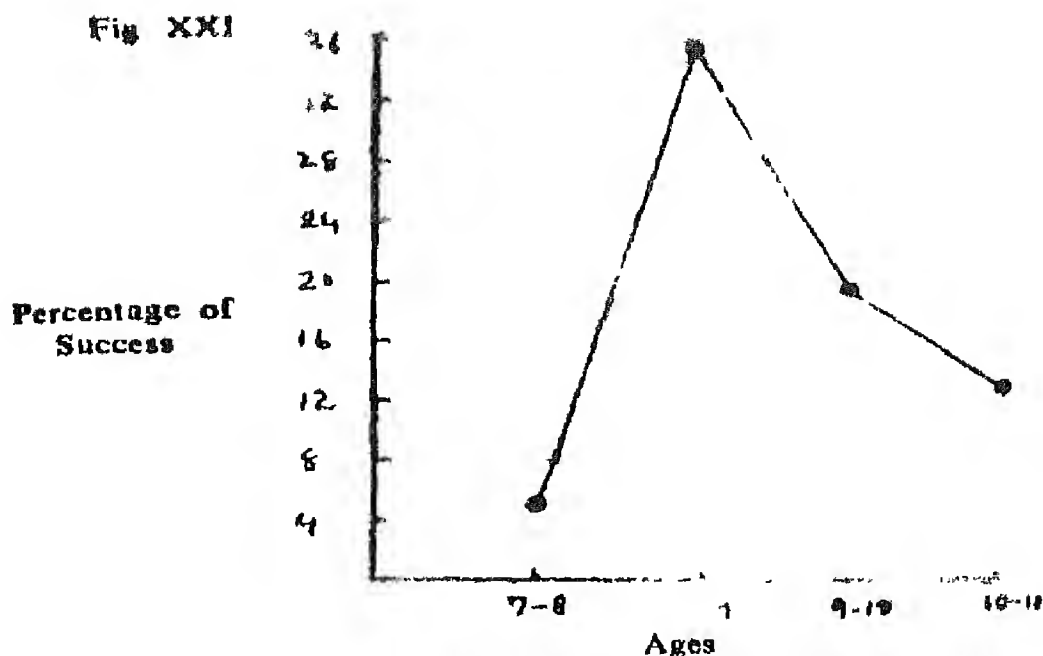
Conservation of Volume (Transitional)
According to Piaget and Inhelder

Fig. XX

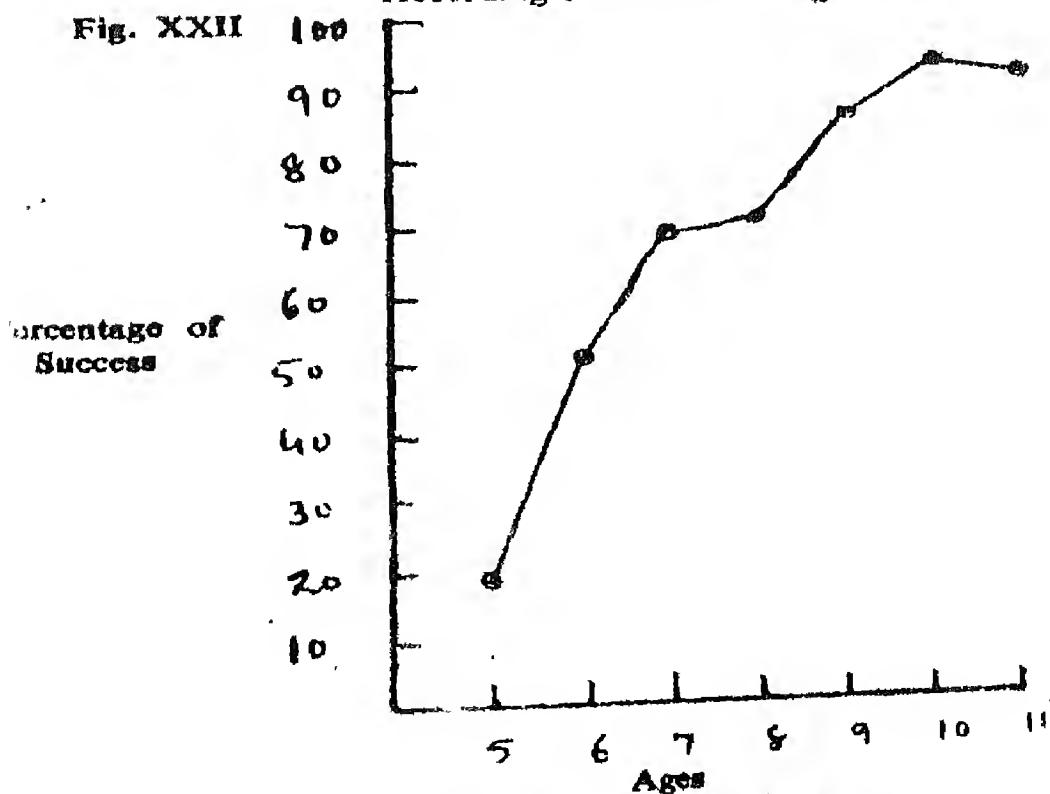
Percentage of
Success



Conservation of Substance (Transitional)
According to Lovell and Ogilvie

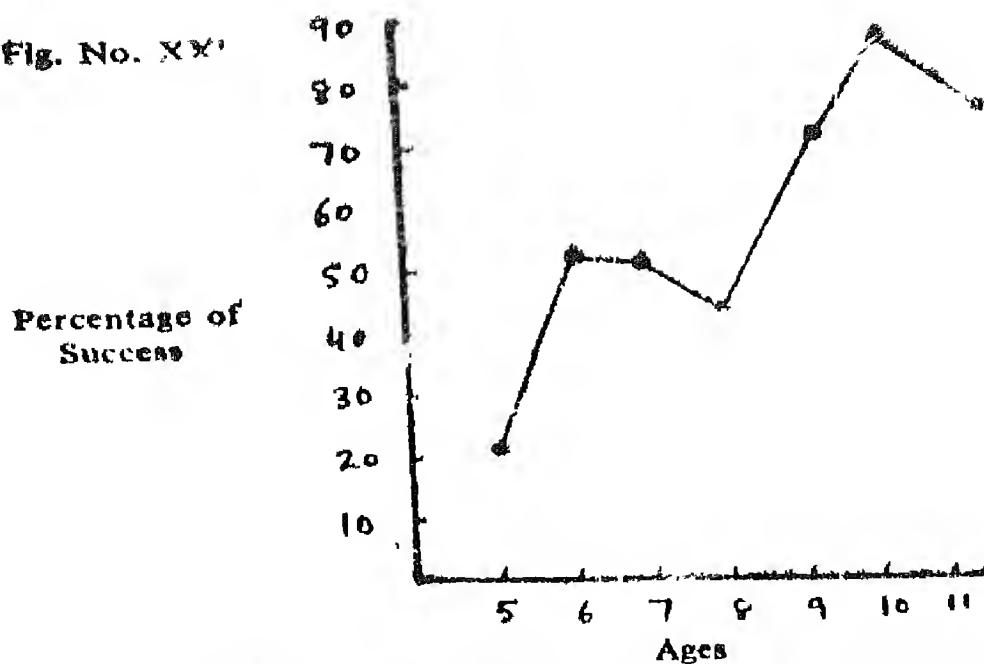


Conservation of Weight (Transitional)
According to Lovell and Ogilvie



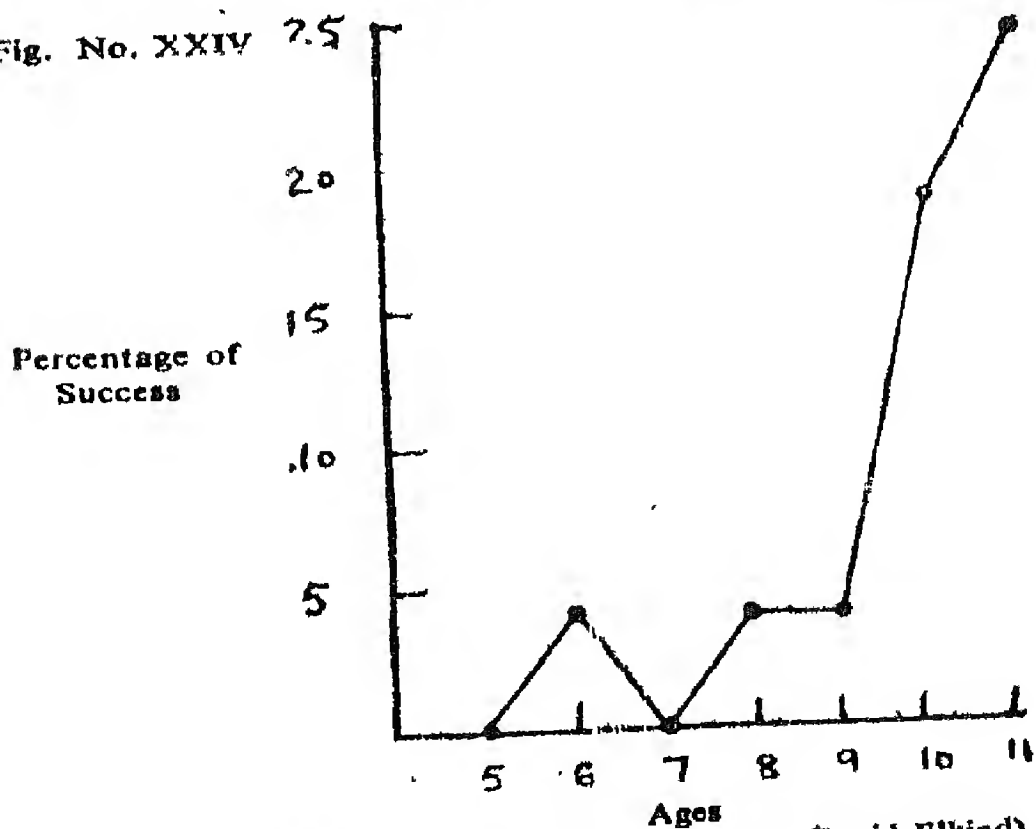
Conservation of Substance
According to David Elkind

Fig. No. XXV



Conservation of Weight (According to David Elkind)

Fig. No. XXIV



Conservation of Volume (According to David Elkind)

Martorano (1977) also encountered this phenomenon on the performance at some Piagetian tasks in her doctoral study 'Developmental Analysis of Performance on Piaget's Formal Operations Tasks' though she could not detect it. The data reproduced from her study regarding the performance at some of the tasks are presented below.

Table VII
Mean Score on Piagetian Tasks at Different Grade Levels (N = 10)
(According to Martorano)

Tasks	Grades			
	6	8	10	12
Rods	2.75	2.95	3.75	3.70
Balance	2.75	2.65	3.40	3.35

The graphical illustrations of the above data are given below.

Fig. No. XXV

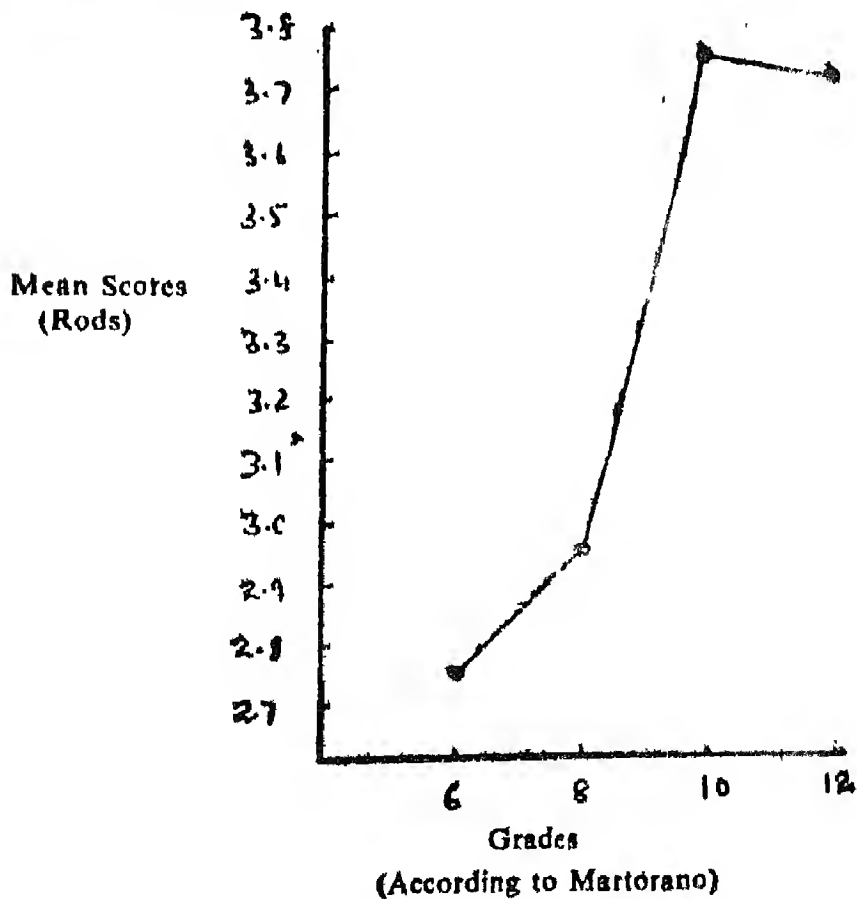
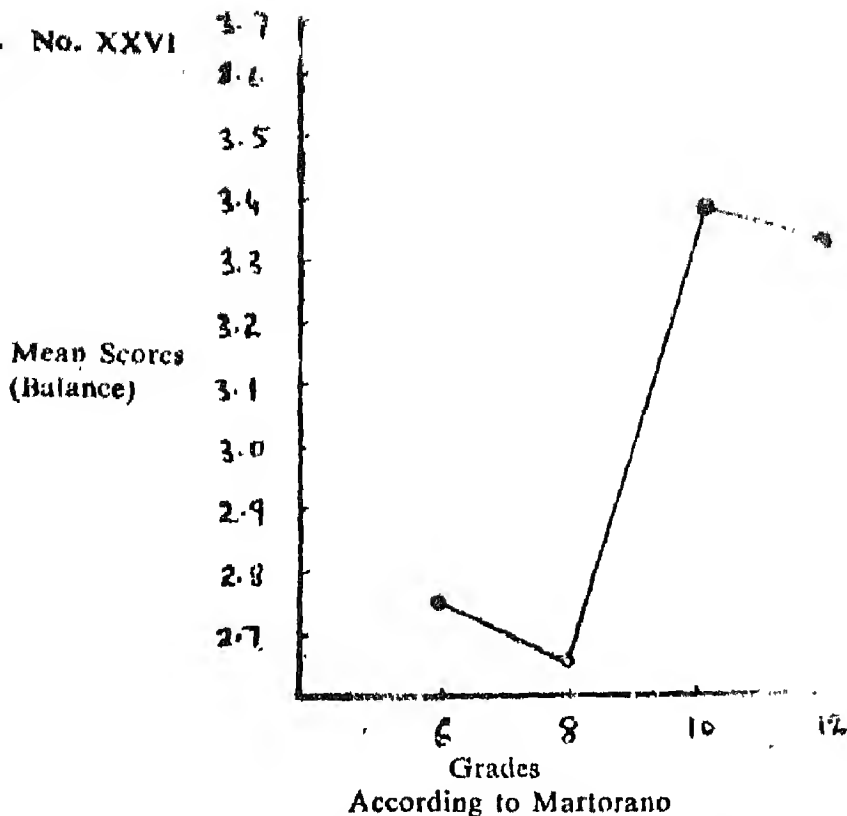


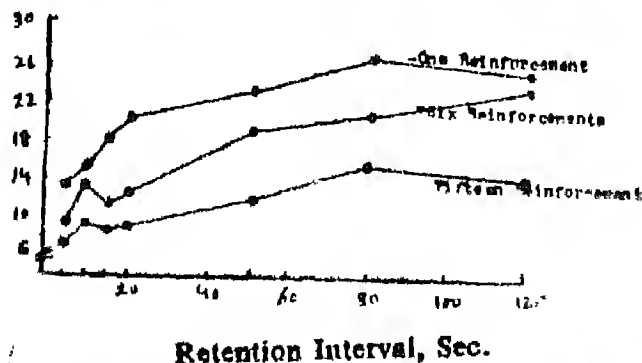
Fig. No. XXVI



It was a firm belief among motor psychologists that motor learning resists forgetting. Recent evidence suggests that part of this generalization has become suspect. It was seen that in short term verbal retention forgetting could occur in a matter of seconds. Adams and Dijkstra (1966) found the same in respect of motor responses : Forgetting is signified by an increase in error and it can be rapid, particularly, when the number of reinforcements is small.

The graphical illustration of the phenomenon is given below.

Fig. No. XXVII



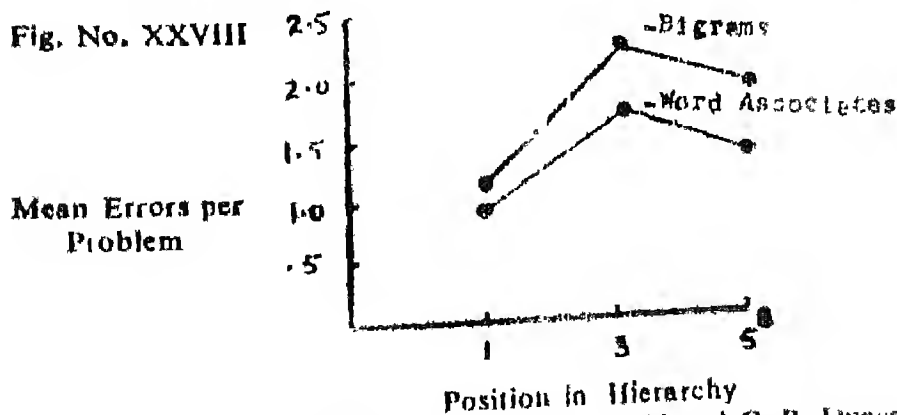
From J. A. Adams and S. Dijkstra (1966) "Short-term Memory for Motor Responses"

It is of sheer interest to note that mean errors per problem, as a function of problem solution in the response hierarchy, also appears to suffer 'hump'. In the context of Words Frequency Effects, Underwood L. Schulz (1960) has suggested a *Spew Hypothesis*, testable in a number of situations, that order of emission of verbal responses is directly related to their frequencies of occurrence. In problem solving, this hypothesis predicts that high frequency of correct response is inversely proportional to the difficulty of the problem. In the context of 'hump effect' it is not unreasonable to reword it as follows : with age as well as low score on individual problems or processes underlying them or both and even the latter three starting from zero as the ascending variables, the frequency of correct score is neither directly nor indirectly related to the problem difficulty, strictly speaking, in the linearity context. While maintaining the increasing positive trend of understanding with age, low correct frequency of correct score at lower ages appears among pupils on problems inhering continuous chain of reasoning which suffers characteristic ups and downs very very similar to humps of camels as judged by their remarkable increase in errors coupled with equally remarkable decrease in errors at higher ages until the correct response firmly settles in individual minds bursting through errors in cross-sectional contexts, with striking drop in frequency of errors, possibly on the development of reversibility in abundance within the context of individual problems or processes.

Two graphical illustrations taken from research literature and given below possibly strengthen a part of the hypothesized phenomenon in entirely different contexts.

Bigram from R. L. Dominowski "Problem Difficulty as a Function of Relative Frequency of Correct Responses"

Word Associates from C. P. Duncan "Response Hierarchies in Problem Solving."



According to R. L. Dominowski and C. P. Duncan

perhaps the same sort of support is received when the hypothesized phenomenon also appears in Serial Rote Learning in the form of a 'bow' when frequencies of errors were plotted for a group of subjects engaged in learning each item in the serial position affect. (Krawiec, 1974) Lastly, Prof. J. S. Bruner when contacted had this to say:

'The type of error that you refer to, which we speak of as growth error, is one in which the growing child tries out a new strategy although it is not well developed and uses it in place of an older one which has been working well. It is errors of this sort which suggest to me the venturesomeness of learning during this early period, the human beings are willing to shift to a less certain and more powerful strategy, before they have it under control, in preference to one which is safe, sound and dull'.

Concluding Statement

On reflection, one is led to conclude that this phenomenon may appear clearly if attacked specifically. In our study, it was not the specific object of investigation. Taking an analogy from optics, polarization of thought during its disappearance is also suspected by authors to suffer from hump. At this stage, no firm explanation is possible because the past history of individual subjects in respect of concepts under study is hardly known for all practical purposes. However, it is possible to suggest the following additional hypotheses for research :

1. Hump Effect appears when thought process moves from a lower stage to a higher stage, the most fruitful area for attack being the transitory period between any pair of the two succeeding stages.
2. It appears at all ages, choice of problems being the determining factor, among pupils at different levels of intellectual development when new concepts are under development. It may appear in sex difference studies relating in achievement and intellectual deterioration among adults later on
3. It is suspected that it may equally appear in the above mentioned contexts when :

(i) Longitudinal studies on thought processes are undertaken.

- Beard, R. M. An Investigation on Concept Formation Among Infant School Children, *Ph. D. Thesis*, Institute of Education, London, 1957.
- Bruner, J. S. et al. A Study of Thinking. *Science Education*, Inc., New York, 1962.
- Bruner, J. S. Personal Communication, 1976.
- Burrack, Benjamin. Methodological Aspects of Problem Solving *Progressive Education*, 1953 Vol. 30 pp. 134-138.
- Busell, G. T. *Patterns of Thinking in Solving Problems*. University of California Press, Berkeley, 1956
- Burt, A. S. The Differentiation of Reasoning Abilities at Adolescence. *Ph. D. Thesis*, Institute of Education, London, 1957
- Carpenter, Horton and Atkin. Quoted from M. W. Travers. *Essentials of Learning*. MacMillan Company Inc, New York, 1963
- Deutsche, J. M. The Development of Children's Concepts of Casual Relations, in R. G. Barker (Ed). *Child Behaviour and Development*. McGraw Hill Book Company Inc, New York, 1943
- Dominowski, R. L. Problem Solving In *Fundamentals and Application of Learning* Ibid,
- Duncan, C. P. Response Hierarchies in Problem Solving. In *Fundamentals and Application of Learning* Ibid, 1967.
- Dunker, Karl. On Problem Solving. *Psychological Monograph*, 1945 Vol. 58 No. 5.
- Durkin, H. E. Trial and Error, Gradual Analysis and Sudden Reorganisation : An Experimental Study of Problem Solving *Arch Psychology*. N. Y. 1937 Vol. 30.
- Elkind, David. Children's Discovery of Conservation. In *The Essential Piaget*. Edited by Gruber and Voneche, Routledge and Kegan Paul Ltd., London, 1977.
- Gruber, Howard, E & Voneche, J. Jacques. *The Essential Piaget*, Routledge and Kegan Paul Ltd., London, 1977.
- Güilford, J. P. *Fundamental Statistics in Psychology and Education*. McGraw Hill Co, New York, 1956.
- Hazlitt, V. Children's Thinking. *British Journal of Psychology*, 1930 Vol. 87 pp. 447-531.

- Heidbreder, B. Problem Solving in Children and Adults. *Journal of Genetic Psychology*, 1928 Vol. 35.
- Hull, Siroke, Haufmann and Kassarin. Quoted from the abridged account given in the *Selected Readings on the Learning Process* by Theodore L. Harris and Wilson E. Schwahn. Oxford University Press, New York, 1961.
- Inhelder, B. On Problem Solving in Paul Henry Mussen (Ed). *Hand book of Research Methods in Child Development*. John Wiley and Sons Inc. 1960 pp. 421-455.
- Krawiec, T. S. The Psychologist. Oxford University Press, London, 1974 P. 215.
- Kruglak, H. Some Behavioural Objectives of Laboratory Instruction. *American Journal of Physics*, 1951.
- Lovell, K. & Ogilvie E. Conservation of Substance : Growth of Conservation of Volume. *In The Essential Piaget* Ibid.
- Maier, Norman R. F. Reasoning in Humans. *Journal of Comparative Psychology*, 1930 pp. 115-143. vol. 10.
- Martorano, Suzanne, C. Developmental Analysis of Performance on Piaget's Formal Operation Tasks, 197
- Marx, M. H. & Bunch, M. E. *Fundamentals and Applications of Learning*. Mac Millan Publishing Co. Inc., New York 1977 pp. 249 and 397.
- Mealings, R. J. Some Aspects of Problem Solving in Science *M. A. Thesis*. University of Birmingham, 1961.
- Mumford, S. C. Factors Involved in Problem Solving with Special Reference to the Problem of Insight. *Ph. D. Thesis*, University of London, 1937
- Muthulingam. An Investigation of Certain Factors in the Physical Science Learning of Secondary Schools in Relation to Aspects of the Achievement, Attitudes and Interests of Fifth Year Pupils. *M. A. Thesis*, Institute of Education, London, 1963.
- Neal, L. A. D. Ed. *Thesis* reported in Science Education, October 1961, pp. 313-320.
- Oakes, M. E. Children's Explanation of Natural Phenomenon, *Teachers and College Contributions to Education*, 1947 No. 926.
- Peel, E. A. Psychology and the Teaching of Science *British Journal of Educational Psychology*, November, 1965.

- Piaget, J. and Inhelder, B. Conservation of Substance, Weight and Volume. In *The Essential Piaget* Ibid.
- Sandhu, T. S. A. Factorial Study of Adolescent Thought Using Piaget Type Tasks. *Ph. D. Study Under Investigation*, 1978.
- Stendler, C. B. Cognitive Development in children and Readings for High School Physics. *The American Journal of Physics*, December, 1961.
- Szekely, L. Knowledge and Thinking and Productive Processes in Learning and Thinking *Acta Psychologica*, 1950 Vol. 7 pp 338-407 and 1 to 24
- Vaidya, N. A study of Problem Solving in Science Among Certain Groups of Adolescent Pupils. *M. A. Thesis*, Institute of Education, London, 1964.
- Vaidya, N. *Problem Solving in Science*. S Chand and Company, N. Delhi, 1968.
- Vaidya, N. *Some Aspects of Piaget's Works and Science Teaching*. S Chand and Company, New Delhi, 1971.
- Vaidya, N. *A Study of Some Aspects of Thinking Among Science Students of Adolescent Age*, Ph.D. Thesis, University of Rajasthan, 1974.
- Vaidya, N. and Rajput, J. S. *Reshaping our School Science Education*. Oxford I B.H. Publishing Co., New Delhi, 1977.
- Vaidya, N. *The Growth of Logical Thinking in Science During Adolescence*. Oxford IBH Publishing Co., New Delhi, 1979.
- Welch, W. W. Review of Research 1968-69 in Secondary Level Science *Journal of Research in Science Teaching*, 1972 Vol. 9 pp. 97-122.
- Wheeler, D. Studies in the Development of Reasoning in School Children. *British Journal of Statistical Psychology*, 1958 Vol. XI, Part II, pp. 137-159.
- Whellock, R. B. An Inquiry into How Far Scientific Method is Gained from Scientific Education. *M. A. Thesis*, Institute of Education, London, 1953

ANALYSIS (v1)

Original data regarding the different Variables

Variables are to be read as follows:

Variable No.	Code	Format	Columns
1.	AGE	F2, 0	7- 8
2.	SEX	F1, 0	9- 9
3.	IV	F2, 0	10-11
4.	TIME	F2, 0	12-13
5.	AGE	F2, 0	14-15
6.	A	F2, 0	16-17
7.	B	F1, 0	18-18
8.	I	F2, 0	19-20
9.	D	F2, 0	21-21
10.	E	F2, 0	22-22
11.	F	F2, 0	23-23
12.	G	F2, 0	24-24
13.	H	F2, 0	25-25
14.	I	F2, 0	26-26
15.	J	F2, 0	27-27
16.	K	F2, 0	28-28
17.	L2	F2, 0	29-29
18.	L3	F2, 0	30-30
19.	L4	F2, 0	31-31
20.	CR1	F2, 0	32-32
21.	CR2	F2, 0	33-33
22.	CL	F2, 0	34-34
23.	GTC	F1, 0	35-35
24.	GAA	F2, 0	36-36
25.	P12	F2, 0	37-37
26.	ELP	F1, 0	38-38
27.	FP2	F2, 0	39-39
28.	ICI	F1, 0	40-40
29.	ST1	F1, 0	41-41
30.	TV	F2, 0	42-42
31.	GRP	F1, 0	43-43
32.	LAN	F2, 0	44-44
33.	AA3	F2, 0	45-45
34.	AA4	F2, 0	46-46
35.	AA5	F2, 0	47-47
36.	AA7	F2, 0	48-48

1 11119266221320803080711071007070812084313452200010671064524749446532102101
2 1111911261131060309090905120805070923102701304008210115651394560213104201
3 1110609200931006080911107111006090627092601203001210826342435260422106202
4 1111920231230304080606080606100408082510540309006201026341444062214102303
5 111071120031008080808100412060810082007030000001001305143465252313108221
6 111060916134081106060911130809101311130615030000200405130494052113106221
7 1110814280860708071010080508120609093716000000005201604343493052113107221
8 11110122104110100905041206121008080719112700000000200713440494546417107211
9 11114151510408060606141004121021206351226040000300405146314745412103303
10 1111416151041008101008080408101410101210001060000200204745465152518105202
11 1123232810205080710101107041204040629115420500003301016047426266424105303
12 111101021143120208100810120808080812271436202000032008060484058416105102
13 11108142804304040103090006020303040422102701300001000903435313442212108304
14 1111702008508110908110905110503020616112711000011010816171535046422105202
15 111141228072070910070902107040810101810090070000321061506356535216104211
16 11110162000507090509110912071109061022132500100009100916357504756316106211
17 11112123306208041214100806060606046241000040000001005250595158127104311
18 111141021143120408061008141210060810221327070100610050585058505247103101
19 11113150850409051207110909012110825090600100002000204548433150214100312
20 1110912161400607100508050604100312122050000501008010703471456350123103211
21 1110607130631109041006121009070908061011260000005200736353524751478703112
22 1111411170841104081011130806090806091404260010005000624956555034117107211
23 11109131709505070808120704090906080715174320000008100726886615460113104121
24 111091923112100505101101508101108101412273000010820042606150571113104121
25 11108141811208060812100808060406100703100900103008010604450645847216105111
26 1111210181021007080611060610120807082208270004006201325350467148312104201
27 111201213061100707120807110708100509311054030200821124404444446112108302
28 11107201006509080911100808101207090825095400501002010725635543453113104131
29 11104091811210090607120604041007060528060011504009120914958716651112103101
30 1110408080710308101007070606060708102508272070100010101473558505010102301
31 11119002005310060711090811090810091007081324095311600106020604862515252324104101
32 11108112410203070906090711060712080716082310301107030016362575052112103302
33 111121318113080605120606131207091010240927000010043303030355474038112103104
34 11118161704100907050709050508031007190826107000062204043474345247104101
35 1110518170630410100904031205020608062608270020210712040565505460113104121
36 1111200240840808091010100407080707113111270901000320205455526165112105101

37 4111513160821105121110806100613081121102500601005020614359625870414106301
38 4111811180050708081010070808090901021120000200003420115350486150418103201
39 111191319062060508140908060906071030095010100220101449675245312104204
40 4111711100720006070711040806080909081706270010010000704736515330113108321
41 111121612120510060908050909101004102611291080010000606057554050113106121
42 41118082006107090806060807101008111134105405000001002051534441302106211
43 11116131506305111108090508060505090822103020300000100503842334142213105102
44 411210819113100506080807081011080810231100101000001005041424140422103102
45 11119131910310040508050808101010081910272210000000705867465667113109102
46 11116212213408090805071007100606100740335451300000101207053604050112107304
47 1111509260020412091010120505080406092407241000000100403057384130422104201
48 11114132608212120912080506080810081020162740504107230905348534052122103201
49 1112021260930608100906110903080709082616151000000001716346334751112103102
50 41115172500208101208121005101404060718132700501100300804953553040213107511
51 111120207408070810101206071208090627143600300000200714250444148324104301
52 4111109080030613071111005080507091221100020600000001033342363312106101
53 111061016084060712080404080410080608140726213000003005057645554400103121
54 41112191602100612090911080604080911140909010000001012044555146514105111
55 11117102809508081012071107120708102408091080000100104744535661122104204
56 41108030612208080809071012090807070849100207000000004052474244422105304
57 111081216114080800061108100808040909210050000000100203339424140312103211
58 411121309103120804080810080609081009120709200000000020508455250416104204
59 11108151908212111012080405080806070420092221500303201004646534030326102102
60 111113181030310121309080611090706122311270150300500605036494030434104101
61 41107132408406090908080705051003070716112710701008101125140434140312103104
62 1110222309407090710080808100806080924084631613107500606441464241222104203
63 4111310201030606081214080607080810101510170200110200725831365747412105302
64 11119131706209100808080408090403090716134241601105710505854695060446103201
65 411071307060050604070406060505060604170627200000040000343314241112103101
66 1110907161040710061110100911050607092207000150000010050248464057117104211
67 41106131810610050810060609070604060914100300200000100504744524630112107311
68 11107103606506070608081110061006080920080501700101100505261504547412105202
69 41108113208312120608101014080804061221090920700000101215752624848323103221
70 1110808190631006040708060906110810082206260070000000513438364051117706211
71 41110051911308091005060909080605030729122220501000000400426744565057112109101
72 11116112610403110911000307071109070935082710100000100205731435551113106111

73	1111111170331109080705070907090605093330409101000000002052525565648312106202
74	111081027125070211100800081211070912191000000000001104738363741322105201
75	11108122611511081112111103090410070929052700300001001574856504112107201
76	1120611150700706070104060206040632111500000010101151515350311210301
77	11206122704212080208060806040800042412080000000000815858625642243106111
78	1120710221031011091210060504060610103208270010001000916767675667524103013
79	1120400181440908130909121109060807062809030000000110514749424645312105104
80	1121700211020404060606040604010404281316000000000071355545404721210304
81	11214141702410101215080406080809061910420100300020150538434414143410504
82	11208091208608041009120006061010111200910009000010061355137422424105104
83	1120812311050605110808020504120404062111270400202005064054425434102101
84	112070615063030612081206100812100610241026020000100115151433050122103101
85	1120608221020090908081109100707108180734030000110812956715160112108201
86	11219242306202060410080608100402040225134131900007611126661576741512103102
87	11217211807411090612131007090507070935064802000008100827158637161511105202
88	1120612240250809081111107101008111033072931100001003224947436142322104102
89	11229182708310120611121107090805160431152722204008610245950496641322104104
90	11221162208207050808090810110905081023045100703006100706166495031416106202
91	112151427123120604101210101060810062912380100005110455451545851414105302
92	112140927110010060608100004020702081810320300106000914945433641444106311
93	1120909181030609100508061306090506091411000000000001048524042212104301
94	1121611231210408100608100812100806081309140000000000303463534151214104121
95	11216131608200110307100705070908140730152100800001100404850544052424107211
96	11109141607506110714090508080909111200928000003000020444049474112111204
97	111814201240812101010940406060407111408390020000011040485460425114102301
98	11108151909410081014100310021114080717102701800009102424751435238416105101
99	111181116062007090908100010110510132109311060110509102258666545418107301
100	1111013200250814090708080806130912081811387030010610072556652561122106103
101	1111714141030705070606091009120511052808480001000210544443575151122103301
102	11117172910507120910151307051007040808104510503200100623545432071312105102
103	111151815103040808080908120608100434135420001003101215152585135212105301
104	111110616122070807110706081008060707230924000000001435162516258112108203
105	1111308261041120512110612061214070514543090001210626245496762422102101
106	11116122103310090405070809040412070618093001000001000352052423842106202
107	11110092010212080606090810081004050620072703000000210614134553741412104201
108	112271919165151214150916150914091511180000000010120030455651503112107201

109	41113131810105111008090908091112130618100910500001000303420474838412104201
110	112030815103060208081012060804040604250814011010002006145505550414101103
111	41215171910406140612120808121212043620532050000401107154616250112106302
112	1120513240610404040804020402020404220800000001100605554665741312107201
113	412191622120041004101212060906081006160836303020030013166507171112105203
114	1120812240830606061012080810060208023415454130000630071585061566222106203
115	412051627125050604100408061006040706310941012000011008154658383122107203
116	1120212201041010041006060608060210063704000000001100105650495058424104101
117	4120811110405030908040305040408021709000010000020304755523840414106311
118	1120508120721102070911061208120805102404180000000200304658566147312104101
119	412061513081040602101020204040406022711184030000210060665266662494103102
120	1121410250820710100308101010060407102111000010000020604052494338112106204
121	4122112221471303061014120808070508032511230010000000207143615048112106204
122	11209142906208090810140608090810101016100130200400515355636167316103111
123	41214172710108120810060406080806140615082600100000100304248495762116106121
124	1121114180340710100608100407050907211024302000060005158667171122104101
125	41215072108309070908071208070508100832110601100002000125142436353113105111
126	11203161608300909071204111109080822080000100000006005146384550112105301
127	412081516084110810100806051210061206140700000007201005250585062112106303
128	112120260210302010001000010002010119084220100001100614748474037426106202
129	4120515230020806020709080610060804102806250000004110915355415045424106122
130	11209062109209080610100903080806081017090000600000200304241384241424103201
131	112111322043070810060910050709070709140404070330600030554655350424104204
132	1120705150312101013101006090508090816041400202002100203435424530422103101
133	412111518062050810080708071009081012231400100010020204843384145112104101
134	11209131008307091213100811080805080723062701001100620404843534747112105101
135	1120706250820508080604031208120814102005080000004205159430404747112104301
136	4121210290821009061314080811060811063707270030100012041624044048436104204
137	4121213130020507091308060408120607092609000110410361031304655052422106301
138	11213132408307080509130708090609100925110020500000120217048554041544103102
139	4121010131141110809090808090506102007450040010013050435556458112104202
140	1120809241030508090807110808060807062308030000100220304303530412103224
141	41212192202107080807080804060602101809061120000220505135413831426105103
142	112121114074091108071207071110607091911331100100132040565560762114109211
143	41209132908308040604060412060608061021102711000102000514767425063217104212
144	41213142811104070706090608061203110633164321200100130206450476450122104201

145 112161025023031207101211081404080804261345209001030000215232605070314100312
146 1121813240830612051011100812081008043515302100010040050554258545416105211
147 112160724063030610040410060612070409310927080000000303630487054212100302
148 1121513220930307081012080308060810380600412000000201414141615641122105302
149 11212100811403081109090800080906081028070000000000003037533030214104201
150 112150703022090807100808101105090810100600100200000004048504730344102301
151 11215131610071008081208050707080909180403040000200704550426060426105301
152 1121108059830406101010001204120408123207030100000000305650576260424104112
153 1121080805044051206060702020904050204060804104000000004151695060412102101
154 112102092309314040611080906101213091126100030400000200815651614057322105304
155 11222030208220811040614001207090305091709001070000700106056485040417104112
156 1121002100109090305081208106070810320810500000000203351424050312104201
157 11215221508107080813080808100808081028075441401006230305850504050436104202
158 1121208181220603080411090704080711082809182020000003004947395151314103101
159 11210715250620807081007000706090508060807000070000101205842464450212104101
160 11210611041120009030710091010605091420040010504002100604159555052112105101
161 11213122413510110809100507120716110803102500002000400315457595060324104101
162 112111131230616040914070807080810081609273130000200825384616150323108211
163 1122517180711206071208120904041207051207544100012740206080646460412104201
164 11210052410012080710071209081210091014060621600000071475344304424104101
165 1121817241231108090808110605070705092504202040000310051448151606434102101
166 112131017083070805090809080809072408264060000100203451525050112104203
167 1121116211031013070910111205091008110904141040002100114783435233312105312
168 11215171411312060412151009080705060817052741100002000404763616172112107304

169 12118192812506080514091406060400040735115401900003200815330334644414102101
170 121181018101080906120810080412121008210909206000034207042256414050216104102
171 12117221807608060607110808071009050832135420801002101716241385237432104201
172 121160924143080810061216081010060806210845100000030006136442404212103104
173 1211315210841106120609041207050303091612510140000200303648335232112106303
174 12110808170050507060909070307060907063009240140000231115550475045432103101
175 121232016022100706080608091206101062014541210100121090454454304212104201
176 12116181410304080711080608080412101018115410900001300914552573052113506221
177 1210909260841110706090809050710080724095431503009222225661546055112105104
178 1211018230930409100611071007090510102011402001006120005054524460422104204
179 121271423114061212100805140708100909220945218040063108155545052213105121

180 421151122093051409110050608090606032911273000206000613058333035324104201
181 1212420240830609110908100411106060833105421308007840906061505758112107304
182 42118222600411070710070309130811070925095431406006041244752574835424104102
183 12121162210308080808081008080408081709151180300401217050655455122105102
184 4211311180840807111108110407110809092510513090400720090445056250422104201
185 12110201414300121308080907090905060625093221102008031225447444040114104202
186 12113101912403041012110410110204120931054800202004000323540495040123106301
187 12119221910312100808101110061004060816114731200005301014948413542112106302
188 42122082908309080511080511050912110722309062080210910031394339434424104112
189 12125192012208060606050503111006101006092700201007110624245515451414106202
190 1213024291251307041411307080708120837105421205006231536070475065313108301
191 42112102207312051012121012070804060919093500201002130922514244437412110201
192 421131212124111008100610060708090711240827011010002011505254445122105301
193 12126131233040210081204080808060432155401204003600814450544854112106301
194 4210816231421400010908080807120806072317540040000300827157504347112106321
195 12118161706204030408100602080206040424135431802007031735854635748515104103
196 421181119085100810101012001110061006180624306000220150494654485112107301
197 1211111220830408100610080010202080023123941201004200203145344142426103201
198 4212231300831003020712060404004060039133741006006731534852635248424105302
199 12119241802207040606100404040200100422112621004002101014438372941414103201
200 4211415190840808100812120406080411083307160140200130050374644053412102101
201 1210614161630608060812040610120004043308470000005110314734405420112105102
202 12116161806305071008080800050202100633113231501002201325138294037323106121
203 1211916241011005101010060406021009226132501300002121125940596367414104112
204 4210714270840705110907110709080808081911300090100400031485050524110200403
205 1210813300307070805040811120809070621114531703609130836148615761412104303
206 1210825260830604100810101060210080822093031102011200416352585050203003321
207 121152323030910101010602120406040827075421104006701217162646058113107321
208 4210523230520511080809050509060807102116010100400361170525546147312106301
209 12117122812112060810120612081208120833303130100031071485852452112101101
210 4211112181450810121010121212060806062314261120200501100458348525216106102
211 121121828063001109050809050813070906180826709000251071292031303112106304
212 1210910251220908121212121112131109164308007100001021101394047341424103104
213 12104172810204080408080408040202060416113601600000100835865704753112106301
214 42109221803110011208100702090907081029105405005003143556260624710707311
215 4211118211010402040406020406050402102115274210200973112625062505113109321

216 12107101705210091011121003090611080620105110400008001035352454347112104201
217 121 610180051011081313061209100511062011273010000510101584044140112106304
218 121082124093080803030708051106091012270854210010510123454641547116107211
219 1210808141030302100812080210040406023012361040220411125142565046444103202
220 1211109170730610080908120604040208102011253020100901013856475251212106301
221 1211312210720305040206010202010402011810271030020611093236384038312103201
222 12122172714102100711060603140408111030105401903005011005466463030112105102
223 121212026102060206091113060511008084614543120470732073585756607117103311
224 1211917311040205080708101111081011091510450150170522100554651304112104201
225 121081725143051108100906081008080808390843113000422030430403303112108202
226 1212112241001404110110508071107090840183600703703220535622475652113107221
227 1211711271021009060508070710120908021209541040705420605250294202113104121
228 12120141913207030807111006091007050517103004010002302145514944112107021
229 12126193308109060810081407120404090422105401008009120015052514041122105301
230 121191525003110906121011100712081010210827030050220404031404047113106321
231 121281526034040905100606110808080911201254000101600715669535450112106312
232 121090920041050911107090508070909132912510050000520080645141646112103201
233 12119141911506090708090710080705103617144521001007500404646504452122106301
234 1211914230831104100908070705121008091812542140009300816164666057514102102
235 121211022102110511121008100811080412211054222050120082545857551412106202
236 12118211812310040410141009061103080323074831401003501404946463737112106301
237 12108162102212020811090006081304061025122730200006201414136366140314106201
238 1210616191131004080909090709120404092507122110100130070364046373113104211
239 1211813271150008041012021107080710051911363030500010022534656466014102211
240 12120202408310060508080606080606120928104650303009000815958395042112107203
241 121202119022110406010121200806071426095461604008000915649664750432104101
242 12117132406400100710060400070812140916133931501000220614053384071416107201
243 12116182608400100607050709050708080914145141103000100214151384048112106101
244 12117091808203080606081000070706090916120010200103200513857433847316108121
245 121061718021005080713110608040808031613094060000001407070627050428103303
246 12113211910120601010805070708100808131151210000021100574044504112103301
247 12117221502207090709130608080511120910180010900102410705551516057414103112
248 12118161707206101006070810090508100823130005000000120805241454741122104301
249 12107171108406120310060810060909091018100910700001200304941514751212104201
250 12121101613211030511100008091108100833120950302100100504841555040413105221
251 12111091500901211005000408090805130826102730100000020304158404030112104302

252 421282123102001207040807110902080909341609513020050204225258606560222104201
253 12117181911212071108061505080809061136112710700500100304955354241414103301
254 421192013122071210100810121008100804371354321090087206165707070112108001
255 12113122708407121010060608080408104130827001000001111052364838322105101
256 4210718240830913060908130909050511020508271020100010100605359557312103201
257 12110151408405080610110808080710060923065031903004311024543564057112104301
258 42118202208411080810080612080810100826132740906007220215752624851414102303
259 421122323103009106080808121303070919082620800001100514758525645412105002
260 12109181810400060810061112060610090433120031508011330914058524241422108304
261 42107182310311060505090810080712060814073311100003140705255604045113105121
262 12112162006105070708080908060607070515075311102000220805654495545112103111
263 4210709240030909041107080814061110052507004020100800604748475652113108321
264 121072038083030506100607071004110506341024200020021003156607225751112103302
265 421231521082110709090610080803070605140822108070000704722494228112104104
266 1210320280950707120911109091212080925081521100701000904743523045112103104
267 42117133710607071008090808060810091354010000420151615846450112105004
268 121092015104110605091009080807070643073910600002401007261494245122102001
269 42118072407204110613090213130907040722141420800700200804730382845113107221
270 121081322122081110090910070710080307231208112017000105053484942451214106114
271 12117102806412060709080811081106081136110011202002320805687626072113104221
272 4212309250820804130912080409120604103014542160000330505961605060112105301
273 12109103214112021206100810120808081014110901300002611206456436453113107211
274 42214132010214081010090708041204081231122710300001230706441516242312104104
275 12207182112214060705120900100914061134122730300002920506248525557422110303
276 1220312180501007140808070708110809063111271140300011211354652346424103101
277 422061222121030608120808060806080813042731506007130515646404052114104101
278 1220408141051010080810081108070713081312131090000700125240456245324105321
279 4220813180931010090607080906080505081509522190000523101524474255117106211
280 12210172208406060910050704060711071046114800300500100914730434053114105001
281 422272318144100608141101206040412083520475200071011127065656062114108312
282 1221612181430007101110120612051011092307270300000240804748535250114109014
283 422091217112006060811090906090608073509271010000000605537566756112108304
284 12207152103070608130813060807090608330652009000200703348565445112104302
285 12217192207070707081207070907080735072721002000601105456546262412105104
286 12220181909212071009070711110508071035090920501002710615284596061413105321
287 42222041110313071106050411070604070920040030300001020303648284045122104203

288 12217162010407080707130612070608101320132610300500020504048405141222106302
289 12207131314105080810021212120611061222102510201001211104240354440112103201
290 12217122112310101214040608080812082110092070000120021434852444112104304
291 1221814160040502100708071115081008103913174010000204000413644444112108503
292 12204151208307071111070606101108090921091500501002101015550486055322105101
293 12209122012108090810100404060408080427162710601000410614555494750112103101
294 122131018094000611101008040808111110311130110200013090513238444112105301
295 1221813160640511090609100809081506083513032000000100405148594048113104311
296 1221112150911209081011070610070506101604090000000020021324138443112106102
297 12221222504209060606160705050306120242085451305006711625265655662448804104
298 12208181210308061008101010080808060828102700202000701026140404344112104201
299 122111434066060706121004050410100827102700800002410615651524052414108301
300 1222416140731060509100914060708110826090903040000330104441535250112105201
301 12218161308203091005100012120705050623061521100007300513641495447112108201
302 12204141508408050804070708070812106120909080200000903840493345214104101
303 122210615081061408100808061408021008220418103000030050455634863214106221
304 12205173210106101008100808060410081407360001003210315347544850112105301
305 12212151510507090612110612060708090823070900102000000604434412031222102311
306 1221421230810402020704030502000020519122741800704330536550555262112107301
307 12210171208404061170081107051105101014112741502004130416560676255212103102
308 12227142110310071012140606060612120626080930600002510726758675267112105304
309 1221615121030906021014040606060404102812091080000311073526257576222106321
310 12218152709300061208100400060507081217082700405100340615852505262247106111
311 1221011181210505100809030908110509081406301000001000504148515258122104301
312 12211102612310120608050406060404062814261030400203081486457435447106301
313 12214062006311060810110710030507050927083641100103100625160635654538104201
314 1220713170640812061204081110160806042415156000001200524848476451412106202
315 12214122510210101008121408061404040424074410702005721005152544138112105301
316 12208142608402100609030812100604080631102440700000100405960515847112108201
317 122191927125130903101216080304101405251254414000044308170707070534103301
318 1221514201140912070809090507090908072407030302003400435143474048212106202
319 122120913103121509070805130805051010170700010010000504235323035112104201
320 12207100802411090708080704040502080424104100400010110634730394442423106302
321 12214112308306051008080508090706050825110000400005010613938565152314102321
322 12223161206303070606080204060408060611125411801108021016040454761436107301
323 12220141904203060609080306080106120714122701101007201035756596858113105812

324	42219152503210080808140402080806100225145041802005100516558657070426104302
325	122080719123080906080806100808100612201020100000100204741434046112105112
326	4221017210810606060206020404060200042310302120000210040395747506141104001
327	42206142106202080406040402060600040620053620901001201113129525852424103102
328	4221016231020806040808060402060004024308334160200231091596150715112106301
329	12217161911310081012090203050409110230114401100002400915366636271112106301
330	4221208240840006101010080806121010041309391190000221092565559566212107001
331	12202072510506101005120805060804080930101600000003100324450713458122106102
332	42210142404306060606081008100408080620102100000001100216371463751117107211
333	422161409123061410080606020809040608250738204020023020044525862261112105203
334	4221514241221005081204080810060408083110382120000120091666344545112106303
335	122071418124141009101203120708106102010180060000311050424238444213105301
336	422091634144080909101108091109151315150809000000110021504424030446106311
337	1222119170620409081106081011060608072209182040000530012545545556226106213
338	42214122710105101210050806020406062609493120101201316683647171112106104
339	1220815231031209101006120506070209291227012000010010414745404217106211
340	4222016290841210061014060408101012082012271100000000303035356515327107011
341	12205121908311030811100708080911007250626411020082002234303135580102101
342	4220917270840710121311080607090711020814001020000000205546524650112108301
343	122131027103081008101410606081214062115190070000000205871715348427106301
344	4221416221140607111107070609080508032110690201001100405130444137414104301
345	1221121280406040206060408080402061029153561503006230727042327070112106202
346	4221016140710805040307020608000503031107090060000511051605548404112103304
347	12202161106310080709090407050809070820082710701010200314344040474116106102
348	4220510140711100808080808080801122309271020010623001505665405112102104
349	12208151910305060710040302070509090725090940300102001224930576058326107301
350	4220910240740503060607060208020611041609544100006220623870526260412104202
351	12207181407305060810040202070507090725105441600107600725553515058112105101
352	42203152312112060810140606100408081236083320207007101325240594765427106111
353	1220906220630910081011100709040704133708270000200020503131344550114105311
354	4220918200021107081011050412071009062910543020210504550545260112105101
355	4221212211220704060908041007051211123175311301106020425756506350422106201
356	42217162413209130811110805110808070818085412301005021106660606271112105201
357	4222724231030708081010080808080810122414270070600403132454565150122103302
358	42215151407107100410091204090611051118135401700104830526371565662112105301
359	1221810301010091111111010080506080626135411401005050704850625757122105101

360 12219131806306141012080604081009060639162801003000650302937375051112104201
361 122120820104070907051203030709060510211742011030032107251515140112105101
362 4221122100810908050910061005080406082713274050200601044741434031412104102
363 42213131708207070709081006090704073413414070500921234695554464/414102104
364 42213121507308090908090709101006090843102641600008301945055595854212107202
365 12210121608305061206110908070506071016122721702005300503641313141112103101
366 422091311107080611100809020510120508080912000004310606056565251412102302
367 1220415130420706060609080706070809070910062150010322050595967636(113107311
368 1221713121210608101312070606090807041509020302002130535041485051117103321
369 4221415141230311090709100510090908043115083080100240406262605763112104801
370 4221706081021103101110080511060515101804094000000000104541475350312103201
371 4220711141400707011305070507071011091410081000000000050434949485(216107301
372 4221209200750911060607090710090708102608090130000010060475515648322106203
373 4221416191420710081008091206090808031211534090000052308244414848422107302

374 131161528103111070706070609050707070717163640000002040424437373048126103201
375 43124112310409091106120613101210080422095331603102320605254555653113105121
376 1311018221221208061211081608101008051814273040000342021505029442044103101
377 43112152008210101410100605080806080426105421501004300245439425134322104201
378 1311614201221005120612100406100410042712545040570302071615035340412103201
379 43118242608214060812100405100408120015075440906003340824758565642112105301
380 43109231805408121009060810090805070810050011503001011902950484237222105304
381 4313129280841010081014121606080808023219096180570906142718171646414104221
382 131262127031110804121016100804061024095440601504740226386626460113108321
383 431121424122140416101612040808100806251554305014080006144350424(414103111
384 43115212900407120708050811081110060729114521203000430414630342037212102104
385 431221528104110406101213090610071206251136310040083408062595358424105111
386 43124152510310030410121410040808100621115342005008750915558515250112106211
387 4311717241041202040814121004081010062510181130001144101455434644112105301
388 1314321361250806041016141208040406023418543210270844070528153351112106111
389 43120202213311070614101509050413110427201831409706230605851464138112104202
390 13128181512308071008111109050510100819150051700311930816686667167212107303
391 43117132007206101010081110060806060814185441204307730205847606154112105303
392 4311915250430904081416100406020612103010442205007921016250574430112107201
393 43116233008207100208070612080810121024164502002004230915954574456112106301
394 43117122010600110906070510071104041412142720502000000624341345650112103201

431 131141722503408100910040412091107070723125450701106500314840385045112106302
432 1311309260721008070310080605090807051808270100000211204350453554112106201
433 13113142103061007091209071103070306210630010000000160504640340112109201
434 1311311270830610081009040410060608092613423010200200205453574558112106101
435 13112091809106041214140907050404080732122701103013120215153453840112104101
436 1312110140611006060616060410080406062215502210500923070475150455112105103
437 13114133002304121006070412110508070819130910400003000005453535550113106221
438 13117151810404100810081206060406081026130930505009230616358645838414104101
439 13119101614305080609121003080810081028175450203107350206671595552122104101
440 131161721074081106091106071012040710231409600031042211047885540522105111
441 1312012260820800808140610140814101032120000401112310413838483737112107302
442 1311411121050809091211100604120804081413431000100100603134495445112105101
443 1311315170840004081007070507070221064401003108300105551645452312106202
444 1311413280761009050808091108090908034112721203011400526359525054106311
445 13109162214507101006100607080502091225143661501503400405352395156114103102
446 13108182207307130811090807090810081032094122205608231316161395131112107101
447 131211728105000900609131010080909052213544050000831051454639475041410111
448 13114121910408101007060606120505080630183420502107310416654616657112103502
449 13110131910205070607120503060808070916125140703109230814255645860112104202
450 1312315190860204020909090605040910042715080140200020725585854463112106201
451 13123161515405090306120709070809130529120620904002301004841393130414106201
452 1311820300931202020814130408041110062209442170120900111616159552112107102
453 1311815211210504100910060604060614062615542210211132052695548485112105301
454 131121308104100708100807071009080710200747114030100105043413141216106201
455 13125172250931103040006140609060510220135142009010341125350645754222103102
456 13117131407301130908050614060507061034183332004013600204755524854312108302
457 13123242211609070510102051102081108261809402070733171556469606112108201
458 13126252708414050805070514060310050628223662209744230006457514248322107301
459 13117221416310060912090806110409071136161800702700200504044553943424104101
460 13114131612508090908101006081008080621083640010115331004341353742412106303
461 1313119251150711091013090707090312062212542120071020904150494840424105302
462 13118182911513040310121111080504130427095312204013720515860575056212104302
463 1313018321431206081412100608110812022105432405004341605548605635216109311
464 131092027121130506061104080811101009401243219000042508160557070222106302
465 131192219141010061114110407080609091915332100170212090494860304112105101
466 13113162412206110811110506101007080528085111602004230703540354244112104301

467 1311720230640308080909080507070707062411543120410334100423552614021717103512
468 13132141611306080908090712080709110412112701400706120504530485135117105211
469 131122223082090904010110906080806351527015010082072525654260112106204
470 1312616220841211080812130906091312063095451805012202936657555461343105311
471 13121132312108080812130909051104100828115321101104100935137534251224107211
472 13112191807307070706071109091212100732105460802506200934029374634112105102
473 1312119300630509008130810070511070623094741307004300743741344847112107301
474 13174221808210100612121004081210111021165431901308401643445392945112102103
475 13132111413210100812121004101008111021105341602509301044857715763123107111
476 13123172408209110608120903060809100512104240600312211236349575050112106201
477 13113151913110100612111004081008091017143820900307111044583453212109202
478 13124181712214061011110060810081110130753420015063016442435114030112102302
479 1311316170831212070912130906091212063809271150030202024439474441110108301
480 13120172012303121010080810141012070729075461600304930734757627156428103102
481 131201622073090507070905110705100620085432203006200715047383437122103102
482 131242529083120608080601006101015082113546170700942737171665950545107303
483 13116222408210090310140906070810071039095432206107001725380504141210006301
484 131762311082101008090907080805050907451354310050070008161614861216105111
485 1311925140040705050412091070508080838112750905004720925652475350216705301
486 131191722100120710101120804080812042315271080200411414952474848113003311
487 1313619171031080212101006080808040221225451600006101215647524850212105104
488 13124141802208100806121004101008100623135441301405401525452374843449102101
489 13116142308310120811060610060810080625155170900003601503030373030312103102
490 1322314220860804061010060808061010082614272220200225115656617267524703113
491 1322215221221008081010080804060609061309544000000630005340536257324104504
492 132252022113120808101114080404060806321254419030095207072647263722448803204
493 13224161410510040812080806080806022511525130401044080556467616114103101
494 13210051308208100804080604120206060417090011100004020404351415542112105204
495 13223163000210040612070509050710110634125401101008400804349396062212105104
496 13217182212410041113140614080507080831135422104009910704336344457222104202
497 1321506090831006081209080706090708091707030000000200504354284438232104201
498 13221171708310111008150605060806120322085411000007221205045525254414706211
499 13212111812308080804160706061012120419095411001507621815634472030422106202
500 1321811080630207100912080611061009112914270120000814122443949514112705101
501 1321607230305071011080605070710040837160010401008140116142374431112107302
502 13226181808103081016080907270314070316165441105005741125962596256424103102

503 4322222170841007101210051210800100839165242263012041206666717171414105203
504 4321909151031208051208080910100812101912033060500753170535561666122103201
505 432100819082100406081008080610081006251144090200825090384242441112108202
506 43209242410403071012080708081408101322135421600004221513842374251347104211
507 4321615210910807081106090608070807062411543200301100825052545042414104104
508 43212182206100612060610040808061007090940401133220614052586558412107201
509 432191220011209070808051007040810063414540140201412071606565546522410522
510 4321413140810120811080608060510080631145040702008100715360484462112104304
511 43220112411210110808040911091111104261509310010413071494145464021710311
512 43223181808112100608100808101206080613065431105011230714880516056432105102
513 43206142112404100704080611060408100816101810203000120615053475542424106302
514 4321813180830910040814091401040806103109273120509030303758606255424104202
515 43213102506405120911080807100608060925062610703006430905650525450216108202
516 432180714043106060814081004081006143412083080400933040525574747427107312
517 4321011181121007080811081102100708103213270000000100905640474040312107101
518 4321408107810505106120604120907111211112000000020041548504442424106102
519 43215141910402021310061010080808061020165210900011210514963554446112109303
520 4321714220941000613081206140706080730091811700011200713948564453127704211
521 4321212231040708121009111090504040827120010501106300405840305141217104111
522 432211720005070508121208060710081214130909302000320062514463041212106302
523 432160915062081210110905100715060712261209000000100403134493131112105202
524 43213102511300603112100612040704062611321160300530060565595657114106303
525 43214102503203080909120912080911071022101810501002300403946514750112103102
526 432192225073101306100813091003080307321645217040156212048717171212109323
527 43215142206303090803060907061011004131021104020000213636413631112106311
528 432181921103100706101010060308100625145121707213200815844454850112106302
529 432191627082060905100908081006081004221414200021152101364754666112210302
530 432222020124101103091011105001107091909271090100533131655503540112105301
531 4321220181220911031011131105001208102007211040100412011385848475444104201
532 4321420251321206051215110706030914073109362160300422091584546505316106311
533 43231232410411020211141212050406090320114840201005220524560544744314104211
534 43232190910410110413101311060009070915162721200000030927083555646122105301
535 43215181410204100807041212040904041015123630302004230725465416154122103201
536 43215201800206050808070510091105040313144221601005720934051304751112108101
537 43224241616310100711110101050808110542052741906007031245760715020114103102
538 4322218240931210071105051008081007081717363100300701083624855565333104211

575 132140720083080700050707040609041107301128115010012105047444040471112104211
576 1321113160911204060709131106060909112015155010200630503430434447316105311
577 132121219593070807121310090409040808172119313010041114034344534314100211
578 1321408070830070910110507100905100738232751801207400504654515455212107301
579 1321010299084001008021008031010080610201145405000040040463647464412104301
580 13217102511400807100503070708090509181325070000762090463844058412105301
581 13211243070306061107080609071109110619195421902004040915850586262434105102
582 13217152710206070611011071009061009191609020010100505340545052214104104
583 1321712130531108080908100806080608083818073000000220813720474431524104101
584 13214071106213101109111407101007061015102720402009200103740555748422104201
585 13210121502300709090903090804071110130630800009110705140474440444404214
586 1322123180721110051209120607040612052610240601209100714720536151317104211
587 13220101607209090708110808081108090932083611106000400935348505454413105111
588 13232282713408061007100808081003100713135412305010042315762586671442102304
589 1321715241231110910060708040613080716140941500000930706363707065112105302
590 1323324161231107071013120610081014061412546170100103100717116668122109311
591 13212102708310040808141008061010100421143610201008121514347475060222102104
592 13221141308312110907150505060807120623104720401006301404954495055414106211
593 13217172508211050812090911131108080921094830500009230513647374840124103111
594 13207122012208091008121110100410121418122730502008430804384405045414104104
595 1321215220109030910071205120904060616173321703011440305181445142312105104
596 1321214160831004100612101012020404081315546170400342050435047574542104304
597 13216141208410080808100610041008080818193620804002300815150385930112110202
598 1321509191040606141006020606081010061510330001008130504967535145132103204
599 13211062708310061008101004060608100416121400600012200515144515350522102104

600 1411192118042081010100610091007101430105320703315501304240434130558803103
601 141190202204060610100808100608080828115321104011700804842453643112103201
602 141112424106100304101214061210081404351154414070077013149634360217108311
603 14115121708204121002080404060204060420090952002006330903641395848113106201
604 14117132310206081208060210081006100633145411903007111103636364440112105301
605 1411172210512061006121210080806101021175402200009510904252475548426105304
606 14132162308609040810120810080806080837232711900009601905555394052426106304
607 141251914104120608061312100906120612310954418085123090485856505212103201
608 1411322280710912110806060408021008042419540110600921813687474043112105202
609 141332224122061204008080610080610061013105471404005401914655555148412105102

[illegible]

646 1412619220920707100608060610050706111754332205112031205246516045313103211
647 14121162200315030510121111071202080425205321801510231105057534240412103301
648 141171426084008061005071104100712112913004000010621015237553732117106112
649 1413321210260705020912101008050610041819546190610906122636165548212103301
650 1412620904305110710131003100502100727165421103003021406245413035112108304
651 14165212512500101008101000091006101038375151706708711216235625745322105301
652 141332518103101106111506060210612092517033210810931425750466262442103204
653 1418032291641406001416160810020812043659547221371708251688686868538103302
654 141151419135051311009090706070505072614501150101111704542384141112103301
655 14124222613310040607091008080507090425312741902110310515535465151113103131
656 141131126064121106061408041210814062722270902306210614854505953113105221
657 141232122145101108121112070602080709210854322017054313263548565226108201
658 14146232614510130609141010605081402223154321090069717371717171434102101
659 1412221231051108070910130607051007124310544190500232072605665051448103201
660 1412523151030703141215040710040809062017272190470703062525550504122103201
661 14121192210409080913080707090907101009105220607009431246666626056113107321
662 1413524301441508041111601040308110315195341908707932030508505358412103101
663 14129222307008070607091110101006100837115462104004801914755435641430103101
664 1413230230311070807070908071209100736135342306504011406862684052112108302
665 1412517200707080910110710090709090732115451302508931705732483843322103201
666 1412820240317080710100606060604080623085462202003021205550415345213108321
667 14139282914412040614100610100610160636140942204509031305957325150112107304
668 14123232400406090809060708060705071037155462205008030524351524232313107211
669 141181817004050608141010100709210092708276090010210081324138323842102101
670 14123230011303060608121109080810030928155421703410831023846484947112105201
671 1412428260031009061106110810040310000718094170650421404138526257113108321
672 1412117191031004081105090070507140429145441501110130424768454051414106121
673 1413020270409080812101006061010140425123652203701730425563565756116107201
674 1413819211051113041110305100513090329205331602007020524470385245112106201
675 14128162212108050810060500090804130621202731900000730904946303045113102121
676 141202321142112020810141608060208140422232731904208051415563455458312105101
677 141332117103100508101010001111108102517544220200384090425648456112104201
678 14112132412213040511121105050907060217143651604005610503838424861414104301
679 14122203211306070606120611110908060840162741905006250723030507070424103102
680 141704210620909071214081203040408063312273060050163031404424031213106322
681 1412221230520909110906111008080711043116273190420563100485554505112103101

682 14143202914214020604141204080408120227184542207003010435549505952434102101
683 1414419171231404080410806020406090918150932206708040925143574448214103102
684 14131222412414020507141208040808100442215461908003051525347314745212105301
685 1414114190920905090605120706030708072518094170570603114131393141424104101
686 1413123291330508120711090707101110630220932206705041226961444041213102102
687 1417330231431210080912130805071080626245462005403070835736523154114105201
688 1413125190631014061308100714001412083221455170500582092494654525414105101
689 1413122231451108081108100407030110092122243090610721090433752545127105312
690 14132232414311040216121408040110120425215451007007391603753403742112104203
691 14127172008410110711090704050910080943142741303703041614248495047412103102
692 14114071512330306040812120806041012042515253050310572100544355473911310411
693 14127262206306080608081010081104080829115120904700630915660493952112105212
694 14114131408410110711090704050910080916092711000606700704760615048112107202
695 1412511221241013070812101210040808043512533030270431092575855656122105104
696 14126131507407040711100904060512110722145222303704621425850524734212106202
697 14132253411414060608151008060810100421435162011703040917163716271112105303
698 14132233212307060806121204060508120544125432205702631234643422948112105201
699 1413202312410110603041306080710100942105451800701730632946294260112105201
700 1412714200831040610141013080412140525113131103705640423930344141213102311
701 14129201807300809100807080709070725122731303701801002920463440112107211
702 1413916171251410080512101007050708062211274140470454050442464760226105102
703 14128162111103081109100804041008080819152741704705030906371594058112104101
704 14173152606205120705080710120608071212164231101702831104941404420424105111
705 14131222010302050807101008090509050838263661902008050816657576142414104204
706 14146262400512101008141204041008140626345432205709821115242596153412106302
707 14143202304060805090309080410100830180942007416012204851485160422104304
708 14258292712300606101210101302071006381436716093067511207064707070426104102
709 1423026170051305131010070905060910063410543230250603082535355424113106321
710 14251282113312070408161210050906120835295431907009071325960646461514104103
711 142221419120040710121014120505081028100901303010320905751504158428104201
712 14228211514207100708100812060805040919115330206009430706444535160422106304
713 14234161507207090413140809110607110735225421904004020907048605164312105103
714 142171418142050907080707110607121024091800602008100716058606245446102101
715 1421617140630408060810060404021206042511241160005711005753605148113106322
716 1423922181051010020608101004041208103517544210005830405263606755222105313
717 1421311220831210060414121008060810061309000010000920080534450473116104201

718 1422018231230061208070810060210120427151871004110330804845434047314106211
719 14217071500300603080909080805091126152741102005120604836363647312104101
720 1422827120030090410100907050805100540235422206003021904456594850317106322
721 1422315151000020410060504081108111216095121400712121704342514251414104101
722 14209120812106101003100812081010100810132531009109051513942463147424103101
723 14226201616404120909041106041006081234165451505009451504654696051212105304
724 1422616170830411006100812060808081027175331704010731105762615447112105204
725 142262023104060908121110090404060804110543120500622170456064504122105204
726 1421713160911014061010040803081306063202700803108001904942453641112109304
727 1423115201021109100810081007020713022412441140530963090556554052417104202
728 14217200900310080606101206120810120831135421506316521615160636152212107202
729 1422120211650412120607050808100610122417122150340823152456374740114104302
730 142262011142001108101010100808060825095332205315521414550375153413104111
731 1421912211531004091210000804112100630060940502002100606040476060112106502
732 1422216170830080613060205041206080723112501200202310504148324643112104301
733 1423120150020312080911014110810131426165322301013270706868605768216106321
734 142131217134120807081207131208030807330927316022033511045322434037444103202
735 1420507251008041212101008101010123409342110001305070405576857426106101
736 142351221004170506120912110501101407161409313011172041447325148426106312
737 14213112117507100502091300080709100427212741301003100213535485451414104201
738 14210182310217080408081212120210120430175122000513130704845566042424104302
739 14223272203120804040810060604080610322335401903006001305350435050436105103
740 14206342510514040810101206060606100540245462205006041103964544538112102301
741 14218252516212060608121208100606120231105461306106301205454593862412105202
742 1420919211460008040812101007040610062178442220850782092566450426106302
743 1420623200310060604100608060210120815165471600204620604457514050122103201
744 14208191613400100810120506100806100620215401900004041004844504052434104113
745 1421119201121004091304100007050700082509545160220900110525440404113103121
746 142232619145070804061214050902081003517544220005240405756326250415104321
747 1421820150750710081207070608021211081509272060100340070615756583424103701
748 14213191400500110809061010060610080813160920601005610715964655435117106311
749 1422511231207041012121008061010003124406140300503003741433042227103111
750 14215232514414050808141406080610080819095461501003740614343474051112104201
751 14206182613310080414070907040308101034131830303005010616464485450424105302
752 14221122510012100616120204120804100738243861005008031004847414540112107101
753 142172417105005070607100711060714110913542220000621122645059545112104201

754 44216163210514060612140804100608140621103630504005030904852515340424105201
755 14222102706205140404120806120713140829092720700002020304133456241346104202
756 44226151812312110111209040605071110401954622075060131303038333424104202
757 1422816300340706000812100806031014063016547170700501122515462525112108202
758 44239222912415020608131608080810120434125442003007020713987504950222105102
759 142261614104061008120808060806081102241600414022505100403957534540214104204
760 4422823160821210060612080906020816043120493220450803050486258557212106302
761 1421019120830010081412080610080604071512152070100110090394043303112104303
762 4421009170820003001211090808110810082302345030100020070394158434414106204
763 142141916161230060808080606100404000823130931503000511015951554047412103104
764 44227152612211050712081205040812090715175471504005430525345485147222103301
765 142182019122101007101208100607100806210731313030043092473042354214103101
766 44226191912612120512101007090706100421090901107108600514548626550217105311
767 14222132812300090408111212080710080414135440603005740833045535450122105101
768 442239131912112100612141006060608120221155440904007600534651586062422107301
769 142152420750908101011120608060407062010183090300122032354855424112104301
770 1421917190630507100910100804050206091812452060600373092414153305113104311
771 1421332191041060710060709100810060825091821605306300524145354535214104211
772 44219212001005041013050808060408102610440130510302062524148515147103211
773 1421516221110070407111305081008120624102741601005510515360383038427107311
774 4422523190661109081212110806050812073112254190300373051624059686414104202
775 1421517190911108081107080707071008092909543100040071031514362565313105311
776 4422321171061006040809120507051212043711543190200582161474657506233104111
777 1423013141040311090510040605040806112812454160200845145048483035412106314
778 142281519003003070710060609121112092615263180250560131355235351436105104
779 142251824104070804060913060803141208181130517010036014254445545124105103
780 44226211510310100607061208140412100620185441900007781106337524050412105304
781 1422913270241208070812140808040212022218185220231065102624060656112105304
782 44216181513308080808110709070410050735105351301002400015152626145212104203
783 14218122216106091209110813040905080623142510800104751314552483845312103302
784 14223131809040603061010100612071510062613274150210704081584858561212105201
785 142232119096102070410120405081010950823132520801200101126030504445213106212
786 44222161906309070811080709090713080528155451603007001615250525745472103202
787 44218202008407091009100511090607160929185341906010041524560706145112104202
788 14242211912410060406160806100506120626142751706005621216544625145322105301
789 4424222211040400041414120610021014063224542220800603092616061122101102

790 14226262914012060810111206060308080620085451303705431223151414741434102302
791 142291725080606014100406040814042005422010700231624350365248117106211
792 142422422145141006071212005030811043320544230470409201555069506112105101
793 14222232314313040814141004100312080827265432007106930834453484745112105304
794 14217102110310050412141008060504160632075441503002030424640525452312105101
795 1422223100507041308090408070810100720063342201004010923956445050312101101
796 142291822411611100511014009050713122113544071000484072505659505424102101
797 14227162208310100808120810040800120619115421203506520824754444330122106201
798 1421710151040610040808060410080406021911092160500403182364048403110104301
799 14240262814314120208121410040504100830175432003003030406166667160416106201
800 14220201510107050708100912090610101038122722202003020505053556756112104202
801 142201816053100606090708081209101113010361190001030914148495150112107302
802 1422419211030611041010008080610120533155441701002400813048495752212105302
803 14213072260841207091109100907050910100909260603604220403045424250123105311
804 142181722510308070813141009060209130825131231404001440703848485052222106202
805 1422008280641109061309060808040910081308363080300300304938525670216107202
806 1421826180831107050812100709040911093415483150200973080603070470212100204
807 142241916063000608090807050302111006441243217024125212155505556140212106203
808 1422312221530810091012080911030607112207410120320336050485045461434104202
809 1421618241410710060613080508080906161554309040073209140433373052112109201
810 142172117102080806140512060808040822103921804403700803450624148112104301
811 14222241514406070206121005090408100817105452203110300804846595450112106302

812 15173282700514110308121200070704120331125452212514031647064646467414102101
813 15141202410508060810141008060608120419135452102214020745854585060416107201
814 151312122021108040913071105021411030175242205209041515348514230112107202
815 15116182500410080714070211070307100922123350803010412314744553041113106331
816 151302230030100809080308110605110626125420060157408262524340113107211
817 15120253006612120612130904070506080441124261702108040923851304150113105221
818 151372025114120506061516031009101022310545140551241103038454040112111302
819 1517825280221208020614081408101012064110546105008921815148403030112108201
820 15136192291251102041209120810031012014019276200611070725644513040112106201
821 151441518050010061013040812041010800155431405116041814462534050112106304
822 1511317270204060711131209080209140617165461501010420724236474430112104202
823 1514422301251406021016140406020608021918276150420561101364156485216106111
824 151341629106110605060906006010612073020542240671308125666066606113104121

825	15124242916510040612151404120612160411225452208016051934251505140414106304
826	1512524280941011090710130607110707101619541200100051734234404050312103201
827	15135262910400061107081009040406071236235251407310051454751535141414104312
828	151382626083000605090908080806100444165442108218061753447344734122102101
829	151331626082001111508050706091101120095430906008721104238363233112103111
830	1513719171030808100410060610081410082915541220801105170454046451212102302
831	15137232512300090811121009060507140829205352209709021026784566267116105211
832	15139242411509070904091209080703090629210942204712560915355626062112103101
833	15132272512410080806121206100204060415265412103710641035853675854113105121
834	1511519281230000408081208100208060407205321604709241124747465054113105121
835	15127192306500040406101006040204060620112732107010931206284535358113105221
836	15116122700406040308080406090507040621132741907012721104943515047113107211
837	15120152108508120808120809070710090917144350801011230813851495147112104201
838	15124181800206040205080604040202060234135331306009551235558564747214104121
839	1511400708306071208040505070510071020120641504006530915143424238213106121
840	151221011103111120110120708110605143110090050200720110434843363112107211
841	1512426230514060605120906100412100433165442207514461725264525242114104211
842	1511920250420506060812130811061005063271344161051231210364336484212105202
843	1513121181210705051010051210050807092712543210611551114836484364212103101
844	151282231105100608120910101004100226160032004516331314938436751112105101
845	1513230230640709050713120410070708073621544220900906210544633533122104301
846	15120142106514060606110906080508100415211720705013230815155384755112104202
847	15131422082007101108100706101005051919533200008221106253624045122105202
848	1513110190808101414060604041210061415331220011443081546458451316104211
849	151922220930005090708091109070612072013543190202154123385043404748102302
850	1516021180960415071203060609071110103022365170601678113458556067536104204
851	15127202112209121066140808040404120826195431608512120804280513058313103111
852	1512805181030807110613080410080608101310270601413790413337473742123105131
853	15132142206207100813111109090609090423184950504300240906034603457312105202
854	15120161007306120110120308101110121213225431908310821614740364445112106102
855	151225331141408011015120208020914043629542241071609183668666506112106102
856	1512421311320902100614160606001010004415485190451040505054476640213105121
857	1512430251240406060810140608041010085370097180971270164566056604112105303
858	1512812101211405111014091410081010012508260220520220503447504734112106201
859	1512718231221407061013120610000812063920544190600501080484340404112105201
860	15117172710300100811080809081110130922125130306002410025085534735212107301

861	1511621201031210080913000507061105041511545180250676052518556505112108231
862	1512315250721005070513091105070807093218544090005631004050475243412103101
863	15143223012710110612141306040708080830295432011508021804748406050112105301
864	1516623251241100512101004050806100454445316125120223057505605112104111
865	15151331896406090911091109030507130639775442210707441715970627058113104311
866	1512521160620411100504091012051009121113546110200513012395746404112104201
867	15122142412300080608141212106060810151754522057066064557166604112107201
868	15119152710311040209140910080606080247095452003504721414860505601010108204
869	1514520240650000414101208020707120834025362002709031425071666257116108301
870	1512421241000508120406040208020606103110502050000573050392029304212104204
871	15130231814510050809100908120503090715174861305006741223966625540112104103
872	15118182410214100208100608081006100323180971502508240524762714557112105301
873	15129122408106081110101210080802080738170942106510050324630403748112107201
874	1512123251441050610111204120410110829120032202007051825034343020112105301
875	1511523291731000414120600040610023513092200501146100496044575201102301
876	15140232407313080410121407060508090226255462203507430634882404130112103102
877	1511915261031051206121308060510050112125441001203650923756605052314104201
878	1512626220641007040511081106001010072424545220207541034240446043112102201
879	151302221062120610061010081402041008241554321057096711258554837314705211
880	15129112102210040804101208041008080829135441903703530904957523640222107301
881	1511033100631208041410101012080606064818542240370113120524537554113706121
882	15117231804307080212060410080602041231155402204005760036641575251010103211
883	15116342010214101012101208060606080827175441709710061324857304240112103201
884	151301528004031204120812040604141208201651621001107515234303340412105202
885	151271726076103091410100209050913042422542160571275132485341404042103202
886	1510226251071007060708071040408120816205401903105761224080994737202104101
887	1511910231230081006110907040709130828172421301003230816030463047413106121
888	1510227231231205051014101007071106072323545180771141313444553144232103301
889	15128242509600081009080811070709040429195240711708061625452513752414104201
890	15129151906312060704140810100610100821162741304708860705446445131414106102
891	1513120311541108041412160006030004022424273140830462093574240446054402303
892	1512613210731007070904081608070811082621271190110774083315146358112105103
893	151292025141080606110808050401060805222209520031073100404149405112104201
894	15126111908405070504090409071011090528191821204404030714357465043112106211
895	1510722331240705031209101204101211072412534170500030604680405440112106203
896	1512317221081304040810140609140910073417006120601020804951575036112106211

897 451461712176101205121508101112061111030180951800511050533454606367312104101
898 15128201510100040812161010310140424212731310013020705146433440212105111
899 451232219065007071214110906080308103221544150700383001666666357112103703
900 152321814125101006101608100608081041710020900007920305930495551412107204
901 4522516220030000807090012070311090800142741801015242014856595152416107202
902 15237212400310070510140907040607120420114560505212031704451625067223104211
903 45231262210506070612080808040807120833125432105007941905544365540412106302
904 15206212000614070214120808090608060838125441606009332025454435541426104103
905 45257281711611070609160607110310110742485442207517092116470707067117100211
906 1523319180050010061008051006040413042820542160431175082504604450414105201
907 45224121810500110711111120907070908110918311403011730715340473450430104311
908 1523820291021400412131412080310060522245462207511071205656445050424104103
909 4523213201061014081210090605081210082313273110101534080575257650122106202
910 1522512201250080812081210061004100824125412003110230926457645057434705303
911 45225132111407141008120810101006030928090002003015231614742535051315106302
912 15218161711410110810060208101008023616481060116231405846495632426105101
913 452372525144140604101410120410100242175452100121481236764675162216106311
914 15217221712404080806140806100808161028193252103005340605157585460442804104
915 1521521181021108060610050812101109092209531040100314040394853546226107212
916 15223142306304100908121214060206121011094030803000210504631504050327105112
917 452200022141207110811090610110709111809101010000200405930374120112105102
918 1522315230731006121207100606110312021707433100220530041543952486226107302
919 452121123073100612120710060610070826131211002010201215431463034426105702
920 15216141600007100408100704081206141034114761400005416724242484040414103202
921 4521302261070600020606030502040412000090940300006320104561627152113108331
922 15218171506203100612090510090909121026214251500003330725431505160112109202
923 152101720004140304031006111081012061208365080100701041635262651442104203
924 1520621241241010071013110907010813052212274100000970001515046505252105304
925 452371921124120503071111209060612071405432105005510715155596171444103102
926 15210191600406080610141306060206160821204941701006003316848415155476103202
927 4522223241241214080612081206040612022511361170200500116830715152546107202
928 15218131810303041008121208120608140621142041403209320516148566640426107202
929 4522211261051204060615121206041010100810244080000810102545266575312107204
930 15215181300407051008091010040204121228093611303007511414748465158224106211
931 4521609210761203090810070610070912062010301030050122042444395740222103202
932 15217192012410080707090410100209006111307485090000720061575044535422105102

969	15236101908710070309130711040505150706015442102706630536064646151524107501
970	15246222308412000307140605070309110622185432202706741336660696057416104202
971	15235192010610060913110607110707130436165130402705730624046495050114107211
972	1523928280631608060414141012041014024414545200501040725263566341422107302
973	15244213114416040408161402040406120228165442100511071316350616051212104201
974	1524226271041204101013120811051010828175142402709011426952694160412104201
975	15273122400312100405101607100406081232104841308204021033740444041312108301
976	152331622122110610121208070406100518103941703010330415750485141112102301
977	15223102710214071007121309080410120105440900408830914248374041312103101
978	15233152514614040208121406080406100222265432106008420715150535240122105301
979	15231172210400031010090510090406120831142762207009050614640585248112105301
980	1523913231040607061410140608061014062419542190001233102404616051314103102
981	1521142506306100708070211090408080926125452103200710934460425137436107201
982	15218182012312060704121512090410100126112741902009720614837503150122104101
983	15237222905614101006141208060204121018083642008007040824054504163422103201
984	152201325094070505091208080807121210619105421704006041115431314131112105301
985	15202182312614070806061206100902080634112830302004340305741515063112104101
986	1520417191150011081009091114040808062314342160010534040603434414112107201